Hands, Knees, and Absolute Space

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I. COUNTERPARTS AND ENANTIOMORPHS

MY left hand is profoundly like but also profoundly unlike my right hand. There are some trifling differences between them, of course, but let us forget these. Suppose my left hand is an exact mirror-image replica of my right. The idea of reflection deftly captures how very much alike they might be, while retaining their profound difference. We can make this difference graphic by reminding ourselves that we cannot fit left gloves on right hands. This makes the point that one hand can never occupy the same spatial region as the other fills exactly, though its reflection can. Two objects, so much alike yet so different, are called *incongruent counterparts.*

In my usage that phrase expresses a relation among objects, just as the word 'twin' does. Thus, no one is a twin unless there is (or was) someone to whom he is related in a certain way. Call this relation *being born in the same birth.* Then a man is (and has) a twin if and only if he is born in the same birth as another. Let us call the relation between a thing and its incongruous counterpart *being a reflected replica.* Then, again, a thing *is* an incongruous counterpart if and only if it *has* one. My right hand has my left hand, and the left hands of others, as its incongruous counterparts. If people were all one-armed and everyone's hand a congruous counterpart of every other, then my hand would not be (and would not have) an incongruous counterpart.

But incongruous counterparthood gets at something further and deeper than twinhood does. No actual property belongs to me because I *could* have been born in the same birth as another. But all right hands do share a property, just because incongruous counterparts of them are possible. Though there are incongruous counterparts of my left hand, it is not their existence that makes it left. It appears to be enough that they *could* exist for my hand to have this property. Let us express this new further and deeper idea by calling hands *enantiomorphs* and by saying that they have *handedness*. Then that is not an idea that depends on relations of hands to other things in space, as incongruous counterparthood does.

We can show this in various ways. A well-worn method is to invent a *possible world*, say, one where all hands are right, as I did a moment ago. A more vivid possible world is one that contains a single hand and nothing else whatever. This solitary hand must be determinate as to being either left or right. It could not be *in*determinate, else it would not be determinate on which wrist of a human body it would fit correctly, that is, so that the thumb points upward when the palm touches against the chest. Enantiomorphism, then, is not a relation between objects that are in space.

II. KANT'S PRE-CRITICAL ARGUMENT

These ideas originate in Kant, of course. Though enantiomorphs crop up in the *Prolegomena*, they feature in and around the *Critique* as illustrating that space is the form of outer sense. But in an early paper,[1] of 1768, "Concerning the Ultimate Foundations of the Differentiation of Regions in Space," he used them to argue for the ontological conclusion that there is absolute space. It is this earlier argument that my paper is all about. Kant rightly regarded his paper as a pioneering essay in *Analysis situs*, or topology, though an essay equally motivated by metaphysical interests in the nature of space. He recognized only Leibniz and Euler as his predecessors in this geometrical field. The argument has found little favor among geometers and philosophers since Kant first produced it. I guess no one thought he was even half right as to what enantiomorphism reveals about space. Kant himself had second and even third thoughts about his argument. My aim is to show that his first ideas were almost entirely correct about the whole of the issue.

This is not to deny that Kant's presentation is open to criticism. He was ignorant of several important and relevant facts which emerged later.[2] But knowledge of these leads me, at least, only to revise the detail of his argument, not to abandon its main structure and content. In the remainder of this section I will set out a version of Kant's argument. I try to follow what I think were his intentions closely and clearly, but in such a way as to indicate where further comment and exposition are needed. Hence this first statement of the argument is rather bald.

[1] G. B. Kerferd and D. E. Walford, trans., *Kant: Selected Pre-Critical Writings* (Manchester: University Press, 1968).

[2] A clear and lively account of many of the facts can be found in Martin Gardner, *The Ambidextrous Universe* (Middlesex, England: Pelican, 1964).

A_1: Any hand must fit on one wrist of a handless human body, but cannot fit on both.
A_2: Even if a hand were the only thing in existence it would be either left or right. (from A_1)
A_3: Any hand must be either left or right (enantiomorphic). (from A_2)

B_1: Left hand and right hand are reflections of each other.
B_2: All intrinsic properties are preserved under reflection.
B_3: Leftness and rightness are not intrinsic properties of hands. (from B_1 and B_2)
B_4: All internal relational properties (of distance and angle among parts of the hand) are preserved in reflection.
B_5: Leftness and rightness are not internal properties of hands. (from B_1 and B_4)

C_1: A hand retains its handedness however it is moved.
C_2: Leftness and rightness are not external relations of a hand to parts of space. (from C_1)

D: If a thing has a nonintrinsic character, then it has it because of a relation it stands in to an entity in respect of some property of the entity.

From A_3, B_3, B_5, C_2, D:
E: The hand is left or right because of its relation to space in respect of some property of space.

Kant places several glosses on E. He claims that it is a hand's connection "purely with absolute and original space" (*op. cit.*, p. 43) that is at issue. That is, space cannot be the Void, a nonentity, since only something existing with a nature of its own can bestow a property on the hand. Again, to have made a solitary right hand rather than a solitary left would have required "a different action of the creative cause" (42), relating the hand differently "to space in general as a unity, of which each extension must be regarded as a part" (37).

The premise D is not explicit in Kant. It is, quite obviously, a highly suspect and rather obscure claim. But I think Kant needed something of this very general nature, for he did not really know what it was about space as a unity that worked the trick of enantiomorphism. It will be possible to avoid this metaphysical quicksand if we can arrive at some clearer account of just what explains this intriguing feature.

III. HANDS AND BODIES: RELATIONS AMONG OBJECTS

A number of relativists have replied to Kant's argument by claiming flatly that a solitary hand must be indeterminate as to its handedness. But it is very far from clear how a hand could *possibly* be neither left nor right. The flat claim just begs the question against

Kant's lemma A, which argues that a hand could not be indeterminate as to which wrist of a human body it would fit correctly. Nevertheless, there is an influential argument to the effect that this lemma is itself a blunder. It is instructive to see what is wrong with this relativist contention.

Kant's reason for claiming that it cannot be indeterminate whether the lone hand is right is perhaps maladroit because it *suggests* that the determinacy is constituted by a counterfactual relation to a human body. The suggestion is hardly consistent with Kant's view that it is constituted by a relation between the hand and space. However, it provides Remnant[3] with a pretext for foisting on Kant an essentially relationist view of the matter. He takes Kant to be offering the human body as a *recipe for telling* whether the lone hand was left or right. He shows quite convincingly that even the introduction of an actual body into space will not let us tell whether the hand is left or right. For suppose a handless body is introduced. The hand will fit on only one of the two wrists correctly, i.e., so that when the arm is thrown across the chest with the palm touching it, the thumb points upward. But this does not settle whether the hand is left or right unless we can also tell whether it is on the left or right wrist that the hand fits. But we can be in no better position to settle this than to settle the original question about the single hand. Possibly we are in a worse position because the human body (handless or not) is not enantiomorphic (except internally, with respect to heart position, etc.). Even if every normal human body had a green right arm and a red left (so that it became enantiomorphic after a fashion), that would not help. For incongruous counterparts of it, with red right and green left arms are possible. News that the hand fits the green wrist enables us to settle nothing about its rightness unless we know whether the body is normal. Settling this, however, is just settling handedness for a different sort of object. Thus, it is concluded, a solitary hand is quite indeterminate in respect of handedness.

What Remnant shows, in these arguments, is that no description in terms of relations among material things or their material parts ever distinguishes the handedness of an enantiomorph. This is an objection to Kant only if his maladroit reason is construed as a lapse into the view that what makes the hand left is its relationship to a body. Since this casts his reason as a *contrary* of the conclusion he draws from it, the construal is improbable. Remnant takes Kant to

[3] Peter Remnant, "Incongruous Counterparts and Absolute Space," *Mind*, LXII, 287 (July 1963): 393–399. It is cited as definitive by Gardner and Bennett (see notes 2 and 8).

have been guilty of the blunder of supposing that, though it is impossible to tell, of the single hand, whether it is left or right, it is quite possible to tell of a lone body, which wrist is left. It would have been a crass blunder indeed. But Kant never says that we can *tell* any of these things. In fact, he denies it. Insofar as Remnant *does* show that relations among the parts of the hand and the body leave its handedness unsettled, he *confirms* Kant's view. His attack on Kant succeeds only against an implausible perversion of the actual argument.

Kant's introduction of the body is aimed, not at showing the hand to be a right hand or a left hand, but at showing that it is an *enantiomorph.* It shows this perfectly clearly. For the hand would certainly fit on one of the wrists correctly. It seems equally certain that it could not fit correctly on both wrists. Whether it fits a left wrist or a right is beside any point the illustration aims to make. Though this expository device may *suggest* a relationist view of enantiomorphism, it does not entail it. It is simply a graphic, but avoidable way of making the hand's enantiomorphism clear to us. The idea that a hand cannot be moved into the space that its reflection would occupy is certainly effective, too. But it is less striking and less easily understood (I shall say more about it later). Kant was quite well aware that this method was also available and, indeed, used a form of it in his 1768 paper.

Thus the objection to the determinacy of handedness in solitary objects fails.

IV. HANDS AND PARTS OF SPACE

John Earman[4] charges Kant with incoherence. No relation of a hand to space can settle handedness. Kant appeals to a court that is incompetent to decide his case. It is useful to look into Earman's objection.

As I understand Earman, he argues as follows: we can plausibly exhaust all relevant spatial relations for hands under the headings of internal and external relations. Kant takes the internal relations to be solely those of distance and angle which hold among material parts of the hand. But these do not fix handedness, since they are invariant under reflection, but handedness is not (B_5). What makes Kant's argument incoherent is that the *external* relations (to the containing space) cannot fix it either. For the external relations can only be those of position and orientation to points, lines, etc., of space outside the hand. (Incidentally, whether these parts of the container space are materially filled or not appears to make no rele-

[4] "Kant, Incongruous Counterparts and the Nature of Space and Space-Time," *Ratio*, XIII, 1 (June 1971): 1–18.

vant difference.) But reflection mimics all such external relations, too. In any case, *changes* in these relations occur only if the hand moves through the space. However, although movement through the space can alter these relations, it cannot alter handedness (C_2). Evidently none of these relations settles the problem. This presents us with an unwelcome parody of Kant's argument: since handedness is not settled by either internal or external relations in space, it must consist in the hand's relation to some further well-defined entity *beyond* space (*op. cit.*, p. 7).

That is an unlucky conclusion. We will have to think again.

Earman offers us a way out of the wood. The spatial relations we have been looking at do not exhaust all that are available. He suggests that we say, quite simply, that *being in a right configuration* is a primitive internal relation among parts of the hand. If we could say that it would certainly solve the problem in a very direct fashion. It would be a disappointing solution since we cannot *explain* the difference between left and right by appeal to a *primitive* relation. But, in fact, there *is* no such relation, as I will try to show in sections VI and VII.

However, my interest in the present section lies in other things that Earman has to say. He frankly concedes that Kant was well aware of the kind of objection (as against the kind of solution) that he offers, so that, in this respect, his criticism is "grossly unfair." For Kant did insist that it is "to space in general as a unity" that his argument appeals. Earman says (8) that he does not see how this helps. But that merely invites us to take a closer look.

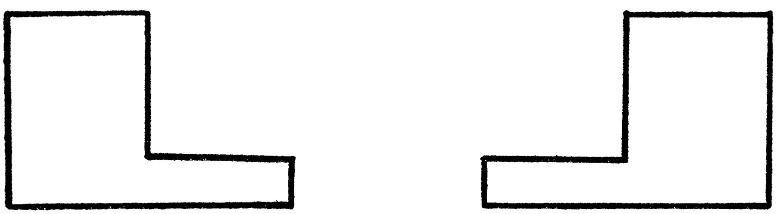
We can get some grip on the idea of "space in general as a unity" by taking a quick trip into geometry. We need not make heavy weather of the rigor of our journey. A *rigid motion* of a hand is a mapping of the space it fills which is some combination of translations and rotations. Such mappings make up an important part of metrical geometry. A reflection is also a mapping of a space, and, like a rigid motion, it preserves metrical features. This last fact, so important for Kant, is easily seen by supposing any system of Cartesian coordinate axes, x, y, and z. A reflection maps by changing just the *sign* of the x (or the y or the z) coordinate of each material point of the hand. In short, it uses the y–z (or the x–z or the x–y) plane as a mirror. Thus it preserves all relations of distance and angle of points in the hand to each other, since only change of sign is involved. Thus the lemma B_1, B_4, B_5 is sound.

Now we can express the idea of enantiomorphism in a new way which has nothing to do with possible worlds or with the relation of one hand to another (actual or possible) hand or body. It does, however, *quantify over all mappings* of certain sorts. We can assert the following: *Each* reflective mapping of a hand differs in its outcome from *every* rigid motion of it. That *is* a matter of space in general and as a unity. (Thus lemma C can be gained virtually by definition.) This quantification over the mappings seems to have nothing to do with any object in the space; not even, really, with the hand that defines the space to be mapped. Though this terminology is much more recent than Kant's, the ideas are old enough. I see no reason to doubt that they are just what he intended. Space *in general as a unity* is exactly what is at issue.

V. KNEES AND SPACE: ENANTIOMORPHISM AND TOPOLOGY

Kant was right. The enantiomorphism of a hand consists in a relation between it and its containing space considered as a unity. This is more easily understood if we drop down a dimension to look at the problem for surfaces and figures contained in them. We need to see how hands could *fail* to be enantiomorphs.

Imagine counterpart angled shapes cut out of paper. They are like but not the same as Ls, since Ls are *directed.* Let me call them *knees*, for short (but also, of course, for the legitimacy of my title).



They lie on a large table. As I look down on two of them, the thick bar of each knee points away from me, but the thin bar of one points to my left, the thin bar of the other to my right. Though the knees are counterparts, it is obvious that no rigid motion of the first knee, which confines it to the table's surface, can map it into its counterpart, the second knee. Clearly, this is independent of the size of the table. The counterparts are incongruous. The first knee I dub left. The second is then a right knee.

Their being enantiomorphs clearly depends on confining the rigid motions to the space of the table top, or the Euclidean plane. Lift a knee up and turn it over, through a rigid motion in three-space, and it returns to the table as a *congruous* counterpart of its mate. That the knees were incongruous depended on how they were put on the table or how they were *in* the space to which we confined their rigid motions.

A different picture is more revealing. Suppose there is a thin vertical glass sheet *in* which the knees are luminous angular color patches that move rigidly about. They are *in* the sheet not *on* it. Seen from one side of the sheet, a knee will be, say, a left knee. But move to the other side of the sheet and it will be a right knee. That is, although the knees are indeed enantiomorphs in being confined to a plane of rigid motions, any knee is nevertheless quite indeterminate as to being a left rather than a right knee, even granted the restrictions on its motion. It could hardly be clearer, then, that nothing intrinsic to an object makes it left or right, even if it is an enantiomorph. Our orientation in a higher dimensional space toward some side of the manifold to which we have confined the knee prompts our inclination to call it left, in this case. It is an entirely fortuitous piece of dubbing. Hence, if the knees were in a surface of just one side (and thus in a *non-orientable manifold*) they would cease to be enantiomorphic even though confined to rigid motions in that surface. (Thus B_3 is correct independently of B_1 and B_2 and despite Earman's objection.)

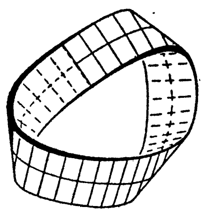
There is a familiar two-dimensional surface with only one side: the Moebius strip. If we now consider knees embedded in the strip, they are never enantiomorphic. A rigid motion round the circuit of the strip (which is twisted with respect to the three-dimensional containing space) turns the knee over, even though it never leaves the surface. However, the Moebius strip might be considered anomalous as a space. It is bounded by an edge. We need the space of Klein's bottle, a closed finite continuous two-space.[5] Rigid motion of a knee round the whole space of Klein's bottle maps it onto its reflection in the space. Knees are not enantiomorphic in this one-sided, nonorientable manifold. They are indifferent as to left or right. Let us say that here they are *homomorphic.*

These general results for knees as 2-dimensional things have parallels for hands as 3-dimensional things. It seems to be pretty clearly the case that, as a matter of fact, there is no fourth spatial dimension that could be used to turn hands "over" so that they become homomorphs. No evidence known to me suggests that actual space is a non-orientable manifold. But it cannot be claimed as beyond all conceivable question that hands are enantiomorphic, and it is not too hard a lesson to learn how they could be homomorphic. So Kant's conclusion A_3, a principal lemma, is false. But this has nothing to do with whether there is one hand or many. It has

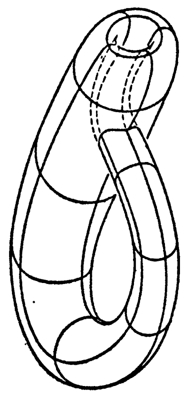
[5] This surface always intersects itself when modeled in Euclidean three-space. But this does not rule it out as a properly self-subsistent two-space. It can be properly modeled in four-space, for example.

nothing to do with an obscure indeterminacy that overtakes a hand if there are no other material things about. It is false because spaces are more various than Kant thought.

Thus, whether a hand or a knee is enantiomorphic or homomorphic depends on the nature of the space it is in. In particular, it depends on the dimensionality or the orientability, but in any case on some aspect of the over-all connectedness or topology of the space. Whether the thing is *left* depends also on *how it is entered* in the space and on how the convention for what is to be left has been fixed. Kant certainly did not see all this. Nevertheless, it should be obvious now how penetrating his insight was.



Moebius strip



Klein bottle

VI. A DEEPER PREMISE: OBJECTS ARE SPATIAL

Clearly enough, Kant's claim that a hand must be either left or right springs from his assumption that space must have Euclidean topology, being infinite and three-dimensional. The assumption is false, and so is the claim. This suggests an advantageous retreat to a more general disjunctive premise for the argument, to replace A_3. Rather than insist that the hand be determinately either left or right, we insist rather that it be determinately either enantiomorphic or homomorphic. Thus, if there were a handless human body in the space, then either there would be a rigid motion mapping the hand correctly onto one wrist but no rigid motion mapping it correctly onto the other; or, there would be rigid motions some of which map it correctly onto one wrist and others which correctly map it onto the other wrist. Which of these new determinate characters the hand bears depends, still, on the nature of the space it inhabits, not on other objects. The nature of this space, whether it is orientable, how many dimensions it has, is absolute and primitive.

What underlies this revision of Kant's lemma A is the following train of ideas. We can dream up a world in which there is a body of water, without needing to dream up a vessel to contain it. But we can never dream up a *hand* without the space in which it is extended and in which its parts are related. To describe a thing as a hand is to describe it as a spatial object. We saw the range of spaces a knee might inhabit to be wide; the same goes for spaces in which a hand might find itself. So dreaming up a hand does not determine *which* space accompanies it, though Kant thought it did. But it does not follow that there could be a hand in a space that is indeterminate (with respect to its global connectivity, for example). We can *describe* a hand, leaving it indeterminate (unspecified) whether it is white or black. But there could not *be* a hand indeterminate in respect of visual properties. Like air in a jar, even an invisible hand can be *seen* to be invisible, so long as we know where to look. (I am here shuffling under a prod, itself invisible in this paper, from David Armstrong.) No considerations mentioned yet admit of a hand that could *be* neither enantio- nor homo- morphic. There seems no glimmer of sense to that expression.

But I spoke just now of there being a wide range of spaces that hands or knees might occupy. What does this mean, and does it offer a route for the relativist between the alternatives I am pushing? What it means is that we can describe a knee, for example, as a mass of paper molecules (or continuous paper stuff) which is extended in a certain metrical 2-space. We can regard this 2-space as bounded by extremal elements that make up edges (or surfaces for the 3-space of a hand). These elements define the shape the mass of matter is extended in by limiting it. A wide variety of global spaces can embed subspaces isomorphic (perhaps dilated) with our knee- and hand-spaces. In short, a kneespace is a *bit* of our ordinary space. (This seems to provide a neat way of defining their identity across possible worlds.) Nothing mooted here is meant to suggest that a hand might somehow be taken from one space to another while being spaceless in the interim. Let us, for a moment, toy with the idea that a knee-space need not be a *sub*space at all, but that it could just end at its material extremities without benefit of a further containing space. Does the hand or knee become indeterminate as to enantiomorphism if we consider it just in its own handspace or kneespace?

Kant evidently feared that it would, and it might seem that he was right. Can't we argue that, for the hand or knee to be either enantiomorphic or homomorphic, there must be enough unified space to permit both the reflection and some class of rigid motions to be defined in it? Otherwise the question whether any rigid motion maps the hand onto its reflection does not have the right kind of answer to yield either result. But this is wrong. What counts is not whether the particular object has a reflection or whether it can be rigidly moved in the kneespace, but whether suitable objects *in general* do. It depends, first, on the *space*, not on the object. Both handspaces and kneespaces are orientable spaces. This is easy to see by imagining a much smaller hand or knee in the space and considering its reflections and the class of its rigid motions in that limited space. Clearly a hand in a handspace is enantiomorphic. This does *not* mean that hands are *intrinsically* enantiomorphic. It means that handspaces are certain kinds of spaces. Any hand must always lie in a handspace *at least* as a *sub*space, but there is no necessity about which space it is a subset of. Nor does there seem to be anything to say about the hand as *material* which could determine even so small a thing as whether the space extends beyond the matter of the hand or is confined to it. Dreaming up a hand means dreaming up a space to contain it whose being and nature are independent and primitive. It is introduced in its own right as a well-defined topological entity.

An oddity here is that, though a hand filling its handspace can be a dilated incongruous counterpart of a small hand in its space, we could never compare or contrast one hand in its own handspace with another in its own handspace. That requires mutual embedding in a common space. I conclude that leftness is *not* a primitive relation with respect to which hand parts are configured. I touch on this topic again in section VII.

But handspaces or kneespaces as anything other than subspaces have, so far, been mere toys of our imagination. The thought that space could just come to an end is one at which the mind rebels. Let us express our distaste for such spaces by calling them *pathological.*[6] I want to do more now than toy with the idea of pathological spaces. It was an important conviction in Kant's mind, I think, that pathological spaces cannot be the complete spaces of *possible* worlds. Though I can think of no strong defense for this deep-lying conviction, which most of us share, I can think of an *interesting* one. At

[6] More technically, I believe that we find a space pathological when it can be deformed over itself to a point, or to a space of lower dimension. Topologists call such spaces contractible [see E. M. Patterson, *Topology* (Edinburgh: Oliver & Boyd; New York: Interscience, 1956), p. 74]. The sphere as a 3-space is deformable through its own volume to a point, but the *surface* of the sphere, as a 2-space, cannot be deformed *over its own area* to a point; so it is not pathological or contractible. The Moebius strip can be deformed across its own width to a closed curve (of lower dimension). It was for this reason that I moved, earlier, to Klein's bottle as a noncontractible proper space.

least, it interests me. Why might one think that space cannot have boundaries?

The thought that space cannot simply come to an end is ancient. The argument was that if, though impossible, you did come to the end, you could cast your spear yet further. This challenge is quite ineffective against the hypothesis that there is just nowhere for the spear to go. But what the challenge does capture, rather adroitly, is the fact that we cannot envisage *any* kind of mechanics for a world at the point at which moving objects just run out of places to go. What would it be like to push or throw such an object? We can't envisage. It would be unlucky if this boggling of the mind tempted us to regard pathological spaces as *contradictory*. To go Kant's way on this is to go the way of synthetic necessary truth. In 1973 the prospects for following that road are dim. But there *are* prospects, more tangible and, perhaps, a bit brighter, since Kripke's semantics for modal logic. It is tempting to pursue this defense of Kant's inference from pathological spaces to global ones which properly contain them. It would be a protracted and rather tangential undertaking, however. The best defense for the disjunction enanthimorphic/homomorphic is to argue, as I did earlier, that hands and knees must be either homomorphic or enantiomorphic whether their containing spaces are pathological or not. This rests the disjunction on the deeper premise that hands are spatial objects and there can be no hand without a space in which it is extended.

Nevertheless, I am inclined to offer the suggestion that, when it is our task to conceive how things might be, *as a whole*, we should ask for what I will call an *unbounded-mobility* mechanics. (I intend the phrase to recall Helmholtz's 'free mobility', which he used to express the possibilities of motion in spaces of constant curvature.) That would rule out pathological spaces for possible *worlds*.

VII. DIFFERENT ACTIONS OF THE CREATIVE CAUSE

So far I have not let the relativist get a word in edgeways. But his general strategy for undermining our argument is pretty obvious. He must claim that there cannot be a space that is a definite topological entity unless there are objects that define and constitute it. Of course, he has to do more than simply to assert this claim; he has to make it stick. That needs at least two things. First, he needs to show how bringing objects into the picture *can* settle topological features of a space in some way—for example, settle the feature that it is an orientable manifold. Then he has to show, next, why only objects can give it. The second of these tasks has the virtue of familiarity, at least, but I know of no discussion whatever of the first.

Let us pick, for our example, the case that still lies before us, of distinguishing orientable from non-orientable manifolds. Suppose we begin with hands or knees in their pathological or limited spaces. These are orientable spaces which can be subsets of non-orientable spaces. How could relations among these hands or knees make some wider space orientable? Knees are simplest. I will talk about them.

Suppose there are two knees, each in its own pathological space, neither being primitively taken as part of a wider, mutually inclusive manifold. The supposition is expressed more accurately, perhaps, in this sentence:

$$(\exists x)(\exists y)(x \text{ is a knee in a kneespace} \cdot y \text{ is a knee in a kneespace} \cdot x \neq y)$$

According to this hypothesis there is *no* primitive spatial relation between the knees. Let us suppose further that some change in one knee causes a change in the other. If cause is the *transmission* of an effect, then this suggestion of a causal connection presupposes that there is, after all, a more primitive spatial relation between the things. But if cause is not the transmission of an effect, then how does the supposition of a causal connection even begin to relate the things in some derivative spatial way? I see no glimmer of light in this sort of approach. We can add knees to knees in this fashion till the cows come home and be no nearer constituting a wider non-orientable manifold, or indeed any wider manifold at all. The trouble is that it is not just our kneespaces that are pathological; *so is the method of adding just described.* What seems always to have been taken for granted is that adding objects is putting them together *in the same unified space.* I do not see how that can fail to mean that we posit *paths* (continua of points) across which they are related (e.g., across which they are at some distance). The reason for always taking that for granted, I guess, is the belief, shared with Kant, that pathological spaces are not possible.

Pathological adding cannot help the relativist. Let us press on with standard adding. A model will help focus our ideas. Suppose there are two strips of paper, one white and the other red. We cut several knees out of the red paper and we intend to embed these in the white strip by cutting appropriate shapes out of the white paper and fitting the knees into them. Clearly, once we have made a cut in the white strip to admit the thick bar of a knee, there are two directions in which we can cut to admit the thin bar.

Well then, a space is orientable if it can[7] be covered by an array of directed entities in such a way that all neighboring entities are like-directed. In our case, the question is whether we can make the

[7] Notice that a space's *non*-orientability is never constituted by how it *is* covered.

white strip orientable or non-orientable by entering the knees in some ways rather than in others. The answer is that these entries have no bearing on the matter at all. Suppose our white strip is joined at its ends to form a paper cylinder. Then we can cover it with knees in such a way as to illustrate its orientability. But clearly this *illustrates* without *constituting* its orientability. That is obvious once we see that we can cut across the strip, give it a twist and rejoin it without changing the way any knee is embedded in it. What we now have is a Moebius strip which is non-orientable. Some pair of neighboring knees will be oppositely directed. That is solely a matter of how the *space* (the white strip) is *pathwise connected globally*. It is quite irrelevant how many knees are embedded in the strip and how their shapes have been cut out for them.

This makes it look, more than ever, as if the space as a definite topological entity can only be a primitive absolute entity; that *its* nature bestows a character of homomorphism, leftness or whatever it might be, on suitable objects. My conviction of the profundity of Kant's argument rests on my being quite unable to see what the relativist can urge against this, except further relativistic dogma. As always, of course, that might mean just that I still have lessons to learn.

But so do relativists. The difficulty of our going to school with open minds on the matter is strikingly shown in Jonathan Bennett's paper "The Difference between Right and Left."[8] His paper is devoted largely to the question whether and how an English speaker whose grasp of the language was perfect, save for interchange of the words 'left' and 'right', could discover his mistake. He has to learn it, not ostensively, but from various descriptions in general terms relating objects to objects. Bennett concludes that the speaker could not discover his error that way, though he could discover a similar error in the use of other spatial words, such as 'between'. It is a long careful, ingenious discussion. But it is an utterly pointless one. Bennett states on page 178 and again on page 180 that in certain possible spaces (some of which we know) there may not *be* a difference between left and right. So how could one possibly discover "the difference" between left and right in terms of sentences that must leave it entirely open whether there *is* any difference to be discovered? That cannot be settled short of some statement about the over-all connectivity of the space in which the things live. The same may be said, moreover, of Bennett's discussion of the case of 'between'. For, given geodesical paths on a sphere (and Bennett is dis-

[8] *American Philosophical Quarterly*, VII, 3 (July 1970): 175–191.

cussing air trips on earth more or less), the examples he cites do not yield the results he wants. The familiarity of relativist approaches appears to fixate him and prevent the imaginative leap to grasping the relevance of those known global spatial results which clarify the issue so completely. No doubt relativism has some familiar hard-headed advantages. But it can also blinker the imagination of ingenious men. It can make them persist, despite better knowledge, in digging over ground that can yield no treasure. Empiricist prejudice is *prejudice.*

Let us get back to my paper strip and the knees embedded in it. It is a model, of course. It models space by appeal to an *object*, one which also defines what is essential to the space. That is, the subspaces, paths, and mappings that constitute the space are modeled by the freedoms and limitations provided by the constraint of keeping objects in contact with the surface of the modeling body (or fitted into it). This modeling body is embedded in a wider true space, which enables the visual imagination to grasp the space as a whole. Realists do treat space just like a physical object in the sense of such analogies. A space is just the union of pathwise connected regions.

The model also shows us a distinction already mentioned, in a clearer light. Whether an object is enantiomorphic or not may depend, in part, on its shape; spheres are homomorphs, even in Euclidean space. It also depends on the connectivity of the space, standardly, in global terms. But, what differentiates a thing which is an enantiomorph from one of its incongruous counterparts is a matter of how it is *entered into the space*—how we cut the hole for it in the white strip. Whether we call a knee in an orientable strip a left knee or a right is wholly conventional; it does not really differentiate the knees themselves at all, but simply marks a difference in how they are entered.

The idea of *entry* is a metaphor, clearly. It springs from our ability to manipulate the knees in three-space, and turn them there so that they fit now one way, now another into the model space. Once *in* the model space they lose that freedom or mobility and are left only that which determines enantiomorphy. It is not easy to find a way of speaking about this which is not metaphorical. But a very penetrating yet not so painfully explicit way of putting the matter is Kant's own, though I believe it to be still a metaphor. The difference between right and left lies in different actions of the creative cause.

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