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Qualitative Spatial Representation and Reasoning with the Region Connection Calculus

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Abstract. This paper surveys the work of the qualitative spatial reasoning group at the University of Leeds. The group has developed a number of logical calculi for representing and reasoning with qualitative spatial relations over regions. We motivate the use of regions as the primary spatial entity and show how a rich language can be built up from surprisingly few primitives. This language can distinguish between convex and a variety of concave shapes and there is also an extension which handles regions with uncertain boundaries. We also present a variety of reasoning techniques, both for static and dynamic situations. A number of possible application areas are briefly mentioned.

Keywords: qualitative spatial reasoning, spatial logics, topology, shape, vague boundaries

1. Introduction

Qualitative Reasoning (QR) has now become a mature subfield of AI as its tenth annual international workshop, several books (e.g. (Weld and De Kleer 1990, Faltings and Struss 1992)) and a wealth of conference and journal publications testify. QR tries to make explicit our everyday commonsense knowledge about the physical world and also the underlying abstractions used by scientists and engineers when they create models. Given this kind of knowledge and appropriate reasoning methods, a computer could make predictions and diagnoses and explain the behaviour of physical systems in a qualitative manner, even when a precise quantitative description is not available or is computationally intractable. Note that a representation is not normally deemed to be qualitative by the QR community simply because it is symbolic and utilizes discrete quantity spaces but because the distinctions made in these discretisations are of particular *relevance* to high-level descriptions of the system or behaviour being modeled. In other words the distinctions are of a *conceptual* nature.

Most QR systems have reasoned about scalar quantities, whether they denote the height of a bouncing ball, the amount of fluid in a tank, the temperature of some body, or perhaps some more abstract quantity. Although there have been spatial aspects to the systems reasoned about, these have rarely been treated with any sophistication. In particular, the multidimensional nature of space has been ill addressed until recently, despite some important early forays such as (Hayes 1985a, Forbus, Nielsen and Faltings 1987).

The neglect of this topic within AI may be due to the *poverty conjecture* promulgated by Forbus, Nielsen and Faltings (Weld and De Kleer 1990, page 562): "there is no purely qualitative, general purpose kinematics". Of course, qualitative kinematics is only a part of qualitative spatial reasoning (QSR), but it is worth noticing their third (and strongest) reason for putting forward the conjecture — "No total order: Quantity spaces don't work in more than one dimension, leaving little hope for concluding much about combining weak information about spatial properties." They point out that transitivity is a vital feature of a qualitative quantity space but doubt that this can be exploited much in higher dimensions and conclude: "we suspect the space of representations in higher dimensions is sparse; that for spatial reasoning almost nothing weaker than numbers will do." However, there is now a growing body of research in the QR and, more generally, in the Knowledge Representation community and elsewhere that, at least partly, refutes this conjecture. A rich space of qualitative spatial representations is now being explored, and these can indeed exploit transitivity.

There are many possible applications of QSR; we have already mentioned reasoning about physical systems, the traditional domain of QR systems. Other workers are motivated by the necessity of giving a semantics to natural language spatial expressions, e.g., (Vieu 1991), which tend to be predominantly qualitative rather than quantitative (consider prepositions such as 'in', 'on' and 'through').[1] Another large and growing application area is Geographical Information Systems (GIS): there is a need for qualitative spatial query languages for example (Clementini, Sharma and Egenhofer 1994) and for navigation (Schlieder 1993). Other applications include specifying the syntax and semantics of Visual Programming languages (Gooday and Cohn 1995, Gooday and Cohn 1996b, Haarslev 1995).

This paper is devoted largely to presenting one particular formalism for QSR, the RCC[2] calculus which has been developed at the University of Leeds over the last few years in a series of papers[3] including (Randell, Cui and Cohn 1992, Cui, Cohn and Randell 1992, Cohn, Randell, Cui and Bennett 1993, Cui, Cohn and Randell 1993, Bennett 1994b, Gotts 1994b, Cohn and Gotts 1996a, Gotts, Gooday and Cohn 1996, Cohn 1995), and indeed is still the subject of ongoing research. This current paper is substantially based on material published in (Cohn, Bennett, Gooday and Gotts 1997) but has been modified so as to be of more relevance to the geo-sciences. Some of the technical detail given in the earlier paper has also been removed.

The rest of this paper is organised as follows. First we motivate the development of spatial representations in which *regions* are the principal entities, and review previous work in this area. Then we present the basic topological part of our *Region Connection Calculus* (RCC) in some detail (although space precludes a full exposition). Following this we extend the calculus with an additional 'convex hull' primitive to allow a much finer-grained representation than a purely topological representation allows. Then we turn to presenting some basic reasoning techniques. Up to this point the representation is in a first-order logic but we explain how a large part of our spatial language can be re-expressed in a zero-order logic to a computational benefit. We also consider a form of temporal reasoning concerning transitions between qualitative spatial relationships. We then describe possible applications of RCC to: characterising geographical features, formulating and interpreting queries within a GIS, simulation of spatial processes and specifying the semantics of a visual programming language. This is followed by a consideration of the relationship between qualitative and quantitative data and the presentation of an extension of RCC to handle regions with uncertain boundaries. We conclude by mentioning some current and future research and summarizing our work.

2. Region-Based Approaches to Spatial Representation

Although the acronym 'RCC' was originally derived from the last name initials of the authors of (Randell, Cui and Cohn 1992), the term 'Region Connection Calculus' is a very apt description of our spatial formalism:[4] the fundamental approach of RCC is that extended spatial entities, i.e. *regions* of space, are taken as primary rather than the dimensionless points of traditional geometry; and the primitive relation between regions — giving the language the ability to represent the structure of spatial entities — is that of *connection*.

There are a number of reasons for eschewing a point-based approach to qualitative spatial representation and indeed simply using the standard tools of mathematical topology. Firstly, regions give a natural way to represent a kind of indefiniteness that is germane to qualitative representations. Moreover the space occupied by any real physical body will always be a region rather a point. Even in natural language, the word "point" is not usually used to mean a mathematical point: a pencil with a sharp point still draws a line of finite thickness! It also turns out that it is possible to reconstruct a notion of mathematical point from a primitive notion of region.

The standard mathematical approaches to topology, general (point-set) topology and algebraic topology, take points as the fundamental, primitive entities and construct extended spatial entities as sets of points with additional structure imposed on them. However, these approaches generalize the concept of a 'space' far beyond its intuitive meaning; this is particularly true for point-set topology but even algebraic topology, which deals with spaces constructed from 'cells' equivalent to the n-dimensional analogues of a (2-dimensional) disc, concerns itself chiefly with rather abstract reasoning concerning the association of algebraic structures such as groups and rings with such spaces, rather than the kinds of topological reasoning required in everyday life, or those which might illuminate the metaphorical use of topological concepts such as 'connection' and 'boundary'. The case against using these standard point based mathematical techniques for QSR is made in rather more detail in (Gotts et al. 1996), where it is argued that the distinction between intuitive and counter-intuitive concepts is not easily captured and that the reasonable desire (for computational reasons) to avoid higher order logics does not mesh well with quantifying over sets of points.

Of course, it might be possible to adapt the conventional mathematical formalisms for our purposes, and indeed this strategy is sometimes adopted (see, for example (Egenhofer and Franzosa 1991, Egenhofer and Franzosa 1995, Worboys and Bofakos 1993)). However, because we take the view that much if not all reasoning about the spaces occupied by physical objects would not, *a priori*, seem to require points to appear in one's ontology, we do not follow this route but rather prefer to take regions as primitive and abandon the traditional mathematical approaches.

In fact there is a minority tradition in the philosophical and logical literature that rejects the treatment of space as consisting of an uncountably infinite set of points and prefers to take spatially extended entities as primitive. Works by logicians and philosophers who have investigated such alternative approaches ('mereology'[5] or 'calculus of individuals') include (Whitehead 1929, Leśniewski 1927-1931, Leonard and Goodman 1940, Tarski 1956, de Laguna 1922) and more recently (Clarke 1981, Clarke 1985) — Clarke developed the the immediate 'ancestor' of RCC — (Simons 1987, Casati and Varzi 1994, Smith 1993). Simons' book contains a review of much of the earlier work in this area.

Because RCC is closely based on Clarke's system, it is worth briefly presenting the main features of this system. Clarke (1981, 1985) presents an extended account of a logical axiomatization for a region-based spatial (in fact Clarke's intended interpretation was spatio-temporal) calculus; he gives many theorems as well to illustrate the important features of the theory. The basis of the system is one primitive dyadic relation $\mathsf{C}(x, y)$ read as "x connects with y."

If one thinks of regions as consisting of sets of points (although we have indicated above that this is not our preferred interpretation), then in terms of points incident in regions, $\mathsf{C}(x, y)$ holds when at least one point is incident in both x and y. There are various axioms which characterize the intended meaning of C (for example, two such axioms state that C is reflexive and that it is symmetric). In Clarke's system it is possible to distinguish regions having the properties of being (topologically) closed or open. A closed region is one that contains all its boundary points (more correctly all its limit points), whereas an object is open if it has no boundary points at all. Many topological relations (for example, regions touching or being a tangential or non tangential part) are defined in Clarke's system and many properties are proved of these relations. Clarke defines many other useful concepts including quasi-Boolean functions, topological functions (interior and closure), and in his second paper provides a construction for points in terms of regions following earlier work by Whitehead (1929). This, however, is faulty; a correction is provided by (Biacino and Gerla 1991).

2.1. Interval Temporal Logics

In placing our work in context, it is important to mention the work done on interval temporal logics for two reasons; first, because the region-based approach to spatial reasoning closely mirrors the interval-based approach to temporal reasoning — they both take extended entities, rather than points, as primitive; secondly, it is, of course, possible to use this work directly by reinterpreting an interval calculus

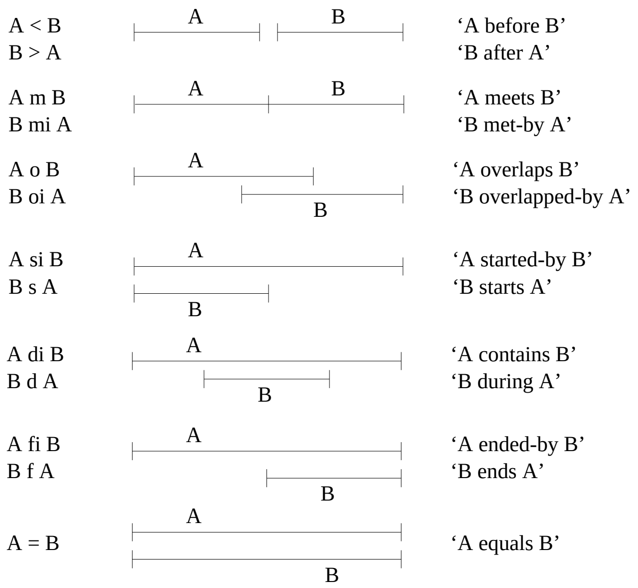

Figure 1. Allen's thirteen interval-interval relations

as a one-dimensional spatial calculus (though, as we shall see, there are problems with this technique).

Allen's interval calculus (Allen 1983) is well known within AI; however, the credit for inventing such calculi is not due to him; Van Benthem (1983) describes an interval calculus, while (Nicod 1924, chapter 2) is probably the earliest such system. Allen's logic defines thirteen *Jointly Exhaustive and Pairwise Disjoint* (JEPD) relations for convex (one-piece)[6] temporal intervals (see Fig.1). The fact that the relations are JEPD means that for any two intervals exactly one of the relations holds, so they provide an exhaustive qualitative classification of possible interval

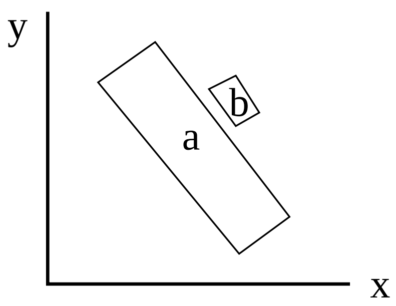

Figure 2. Illustration of the inadequacy of describing two-dimensional relationships in terms of Allen's interval relations in each dimension

relations. Various authors including Mukerjee and Joe (1990) have used Allen's system for spatial reasoning, using a copy of the calculus for each dimension and associating a multi-dimensional object with its projection onto each axis. However, although attractive in many ways, this has the fundamental limitation that it only works correctly for rectangular objects aligned to fixed axes. Consider the configuration in Fig.2: the two rectangles are not so aligned, and although the smaller one is part of the larger one when projected to each axis individually, this is not so in two dimensions; but this cannot be detected by comparing the one-dimensional projections.

3. An Introduction to the Region Connection Calculus (RCC)

The original motivation for this work was an essay in Naïve Physics (Hayes 1985b, Hayes 1985a), We were interested in developing a theory for representing and ultimately reasoning about spatial entities; the theory should be expressed in a language with a clean well-understood semantics. Our desire was principally to create an epistemologically adequate formal theory (rather than necessarily a cognitively valid naïve theory).

We should make precise exactly what counts as a region. In our intended interpretation the regions may be of arbitrary dimension, but they must all be the same dimension and must not be of mixed dimension (for example, a region with a lower dimensional spike missing or sticking out is not intended). Such regions are termed *regular*. Normally, of course our intended interpretation will be 3D, though in many of the figures in this paper, for ease of drawing, we will assume a 2D world (as is also usual in GIS applications). We will deal with the question of whether regions may be open, closed or both below. We also intend regions to *really* be spatially extended, i.e. we rule out the possibility of a region being null. Other than these restrictions, we will allow any kind of regions, in particular they may be multi-piece regions, have interior holes and tunnels.

Our initial system was reported in (Randell and Cohn 1989), which followed Clarke's system closely. However, in (Randell, Cui and Cohn 1992) we presented a revised theory that deviates from Clarke's theory in one important respect, which has far-reaching implications. The change is to the interpretation of $\mathsf{C}(x, y)$: Clarke's interpretation was that the two regions x and y share at least one point whereas our new interpretation is that the topological closures of the two regions share at least one point. Because we consider two regions to be identical if they are connected to exactly the same set of regions, so we could regard regions as equivalence classes of point-sets whose closures are identical. We also, require regions to be of uniform dimension and in terms of point-set topology this means that all the sets in these equivalence classes should have *regular* closures. From within the RCC theory it is not possible to distinguish between regions that are open, closed or neither but have the same closure, and we argue that these distinctions are not necessary for qualitative spatial reasoning. Such regions occupy the same amount of space and, moreover,there seems to be no reason to believe that some physical objects occupy closed regions and others open, so why introduce these distinctions as properties of regions? But Clarke's system has the odd result that if a body maps to a closed region of space then its complement is open and the two are disconnected and not touching! Another peculiarity is that, if a body is broken into two parts, we must decide how to split the regions so formed: one will have to have be open (at least along the boundary where the split occurred) whilst the other must be closed and there seems to be no principled reason for this asymmetry.[7] Thus we argue that, from the standpoint of our naïve understanding of the world, the topological structure of Clarke's system is too rich for our purposes, and in any case appearing in this formal theory, it poses some deep conceptual problems. Furthermore, is it necessary to understand sophisticated topological notions such as interior and closure to create a theory of 'commonsense' qualitative space?

It should be noted that the absence of the open/closed distinction from our theory does not make it incompatible with interpretations in terms of standard topology. A particularly straightforward model is that the regions of our theory are the (non-null) elements of the *regular open Boolean algebra* over the usual topology on $\Re^n$. In such an algebra the Boolean product operation is simply set intersection, while Boolean complement corresponds to the interior of the set complement (hence, by DeMorgan, the (regular open) Boolean sum of two (open) sets is the interior of the sum of their closures). Thus all regions are identified with regular open sets.[8] We may then say that two regions are connected if the closures of the (regular open) sets identified with the regions share a point. So, although openness and closure figure in the model theoretic interpretation of the theory, they are not properties of regions and indeed have no meaning within the theory itself.

Hard-line critics of point-based theories of space might still argue that giving a point-set-theoretic semantics for our theory of regions is unsatisfactory. However, classical topology can be formulated in a purely algebraic framework,where the point-set interpretation is not essential (McKinsey and Tarski 1944). An alternative interpretation of C might be given informally by saying the distance between the two regions is zero. To do this formally would obviously require some (weak) notion of metric space definable on regions but we have not yet attempted to formally specify a semantics of this kind.

Insofar as openness and closedness are not properties of our regions, our theory is simpler than theories such as Clarke's, and hence, we believe that it will also prove to be more suitable for computational reasoning. Furthermore, we believe that the loss of expressive power resulting from our simplification does not restrict the utility of our theory as a language for commonsense reasoning about spatial information. It might be argued that without the open/closed distinction, certain important types of relation between regions cannot be differentiated. For example, Asher and Vieu (1995) have distinguished 'strong' and 'weak' contact between regions. In the former case the regions share a point, whereas in the latter they are disjoint but the closure of the 'topological neighbourhood' of one region is connected to the other. Two bodies may then said to be 'joined' if the regions they occupy are in strong contact but merely 'touching' if their regions are in weak contact. Whilst we acknowledge that the distinction between bodies being joined and merely touching is important, we believe that these relations are not essentially spatial and therefore should not be embodied in a theory of spatial regions. They should rather be modeled within a more general theory of relationships among material substances, objects and the regions they occupy.

To formalize our theory we use a sorted first-order logic based on the logic LLAMA (Cohn 1987), but the details of the logic need not concern us here. The principal sorts we will use are Region, NULL, and PhysOb. Notice that with this sort structure we distinguish the space occupied by a physical object from the physical object itself, partly because it may vary over time which we represent via a function $\mathsf{space}(x,t)$.[9] The sort NULL is true of regions that are not spatially extended and is used to model the intersections of disjoint regions or the spatial extent of physical objects that do not exist at a particular time for example.

In fact, the axiomatic theory we have developed so far deals only with relationships between entities of sorts Region and NULL. Axiomatization of relations involving physical objects would be part of the more general theory of material substances in space, which was mentioned above. So, at present, the sort PhysOb and the $\mathsf{space}(x,t)$ merely serve to indicate how our theory would be incorporated into this much broader theory.

3.1. Axioms For C

Since our interpretation of C has changed, we need to re-axiomatize it and redefine many of the relations Clarke defined which we still want to use. The two main axioms expressing the reflexivity and symmetry of C in fact remain unchanged:

$$\forall x[\mathsf{C}(x,x)] \tag{1}$$

$$\forall x \forall y[\mathsf{C}(x,y) \rightarrow \mathsf{C}(y,x)] \tag{2}$$

Using $\mathsf{C}(x,y)$, a basic set of dyadic relations are defined (Randell, Cui and Cohn 1992, section 4). Definitions and intended meanings of those used here are given in table 1. Unless otherwise specified, the all arguments to the functions and predicates we define are of sort Region. The relations P, PP, TPP and NTPP being non-symmetrical support inverses. For the inverses we use the notation $\Phi\mathsf{i}$, where $\Phi \in \{\mathsf{P}, \mathsf{PP}, \mathsf{TPP}, \mathsf{NTPP}\}$, for example, TPPi. Note that the definition of overlap (equation (7)) ensures that connection and overlap are different: if two regions overlap then they share a common region, while this need not be the case for connecting regions, which need only 'touch'.

Of the defined relations, those in the set $\{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{EQ}, \mathsf{TPP}, \mathsf{NTPP}, \mathsf{TPPi}$ and $\mathsf{NTPPi}\}$ (illustrated in Fig.3) are provably JEPD (Jointly Exhaustive and Pairwise Disjoint). We refer to this set of eight relations as RCC8. The complete set of relations described above can be embedded in a relational lattice. This is given in Fig.4. The symbol $\top$ is interpreted as tautology and the symbol $\perp$ as contradiction. The ordering of these relations is one of subsumption with the weakest (most general) relations connected directly to top and the strongest (most specific) to bottom. For example, TPP implies PP, and PP implies either TPP or NTPP. A greatest lower bound of bottom indicates that the relations are mutually disjoint.

Table 1. Some relations definable in terms of C

	Relation	*interpretation*	*Definition of* $R(x, y)$
(3)	$\mathsf{DC}(x, y)$	x is disconnected from y	$\neg\mathsf{C}(x, y)$
(4)	$\mathsf{P}(x, y)$	x is a part of y	$\forall z[\mathsf{C}(z, x) \rightarrow \mathsf{C}(z, y)]$
(5)	$\mathsf{PP}(x, y)$	x is a proper part of y	$\mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x)$
(6)	$\mathsf{EQ}(x, y)$	x is identical with y	$\mathsf{P}(x, y) \wedge \mathsf{P}(y, x)$
(7)	$\mathsf{O}(x, y)$	x overlaps y	$\exists z[\mathsf{P}(z, x) \wedge \mathsf{P}(z, y)]$
(8)	$\mathsf{DR}(x, y)$	x is discrete from y	$\neg\mathsf{O}(x, y)$
(9)	$\mathsf{PO}(x, y)$	x partially overlaps y	$\mathsf{O}(x, y) \wedge \neg\mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x)$
(10)	$\mathsf{EC}(x, y)$	x is externally connected to y	$\mathsf{C}(x, y) \wedge \neg\mathsf{O}(x, y)$
(11)	$\mathsf{TPP}(x, y)$	x is a tangential proper part of y	$\mathsf{PP}(x, y) \wedge \exists z[\mathsf{EC}(z, x) \wedge \mathsf{EC}(z, y)]$
(12)	$\mathsf{NTPP}(x, y)$	x is a nontangential proper part of y	$\mathsf{PP}(x, y) \wedge \neg\exists z[\mathsf{EC}(z, x) \wedge \mathsf{EC}(z, y)]$

For example with TPP and NTPP, and P and DR. This lattice corresponds to a set of theorems (such as $\forall xy[\mathsf{PP}(x, y) \leftrightarrow [\mathsf{TPP}(x, y) \vee \mathsf{NTPP}(x, y)]]$) which we have verified.

Clarke axiomatized a set of function symbols in terms of C; the topological ones (interior, exterior, closure) we omit since (as already discussed) we do not wish to make these distinctions. However, he also defined a set of quasi-Boolean[10] functions which we will also require, though our definitions differ. The Boolean functions are: $\mathsf{sum}(x, y)$, the sum of x and y; $\mathsf{compl}(x)$, the complement of x; $\mathsf{prod}(x, y)$, the product (intersection) of x and y; and $\mathsf{diff}(x, y)$, the difference of x and y (that is the part of x that does not overlap y); and the constant, u, the universal region. For brevity we will often use $*$, $+$ and $-$ rather than prod, sum and diff. The functions: $\mathsf{compl}(x)$, $\mathsf{prod}(x, y)$ and $\mathsf{diff}(x, y)$ are partial but are made total in the sorted logic by specifying sort restrictions and by letting the result sort of the partial functions be $\mathsf{REGION} \sqcup \mathsf{NULL}$. The quasi-Boolean functions obey appropriate axioms which can be found in (Randell, Cui and Cohn 1992) and also in (Cohn, Bennett, Gooday and Gotts 1997).

As already mentioned, and will be clear from the fact that we have introduced the sum function, regions may consist of disconnected parts. We can easily define a predicate to test for one-pieceness:[11]

$$\mathsf{CON}(x) \equiv_{def} \forall yz[\mathsf{sum}(y, z) = x \rightarrow \mathsf{C}(y, z)] \tag{13}$$

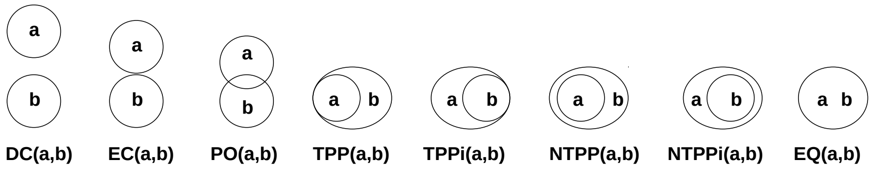

Figure 3. Illustrations of eight JEPD relations

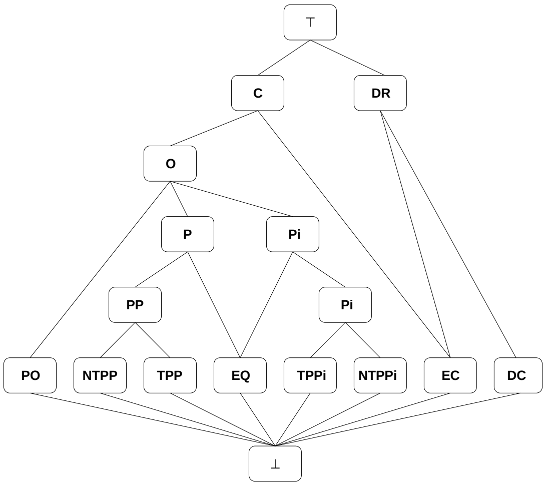

Figure 4. A subsumption lattice of dyadic relations defined in terms of C

Clarke's theory stipulates that every region has a nontangential part, and thus an interior (remembering that in Clarke's theory a topological interpretation is assumed) and is essential to ensure that the definition of $\mathsf{P}(x, y)$ works as intended. Although RCC does not suffer from this problem we do include the axiom

$$\forall x \exists y[\mathsf{NTPP}(y, x)] \tag{14}$$

The consequences of not having this axiom are explored in (Randell, Cui and Cohn 1992) and further in (Gotts 1996a) where atomic multi-region models are shown not to exist. Alternatively, if an axiom is included to rule out the model consisting of a single atomic region, then the formula above need not be an axiom since it would follow as a theorem.

Clearly, if every region has a non-tangential proper part, then in every model there will be an infinite number of regions. However, this is not in itself a problem. In the logical approach to spatial representation, we deal with formal expressions describing types of spatial situation. We do not represent the structure of these situations directly. Thus although our theory ensures that space is infinitely divisible, this does not mean that an implementation of our reasoning system would require infinite data structures. On the contrary, our representation of complex situations in terms of a set of high-level qualitative facts will often be very concise.

3.2. Theorems of RCC

In (Randell, Cui and Cohn 1992) we cite a number of important theorems which distinguish RCC8 from Clarke's system. First, note that for Clarke, two regions x and y are identical iff any region connecting with x connects with y and vice-versa (this is an axiom of *extensionality* for C), that is

$$\forall xy[x = y \leftrightarrow \forall z[\mathsf{C}(z,x) \leftrightarrow \mathsf{C}(z,y)]] \ . \tag{15}$$

This is a theorem of RCC.[12] In the new theory, an additional theorem concerning identity,

$$\forall xy[x = y \leftrightarrow \forall z[\mathsf{O}(z,x) \leftrightarrow \mathsf{O}(z,y)]] \ , \tag{16}$$

(extensionality in terms of O) becomes provable, which is not a theorem in Clarke's theory: any region z which overlaps a closed region x will also overlap its open interior (and vice versa), thus making them identical according to this axiom, but Clarke distinguishes open and closed regions so they cannot be identical, thus providing a counterexample.

Perhaps the most compelling reason that led us to abandon Clarke's semantics for C is the following theorem expressing an everyday intuition about space, that, given one proper part of a region, then there is another, discrete from the first:

$$\forall xy[\mathsf{PP}(x,y) \rightarrow \exists z[\mathsf{P}(z,y) \wedge \neg\mathsf{O}(z,x)]] \ . \tag{17}$$

This is provable in the new theory, but not in Clarke's: the interior of a closed region is a proper part of it, but there is no remaining proper part, since in Clarke's (and our) system the boundary of a region is not a region. A related theorem is the following:

$$\forall xy[\mathsf{PO}(x,y) \rightarrow [\exists z[\mathsf{P}(z,y) \wedge \neg\mathsf{O}(z,x)] \wedge \exists w[\mathsf{P}(w,x) \wedge \neg\mathsf{O}(w,y)]]], \tag{18}$$

which again is a theorem in the new theory but not in Clarke's. A counter-example arises in Clarke's theory where we have two semi-open spherical regions, x and y (with identical radii), such that the northern hemisphere of x is open and the southern hemisphere is closed, and the northern hemisphere of y is closed and the southern hemisphere open. If x and y are superimposed so that their centers and equators coincide, then x and y will partially overlap, but no part of x is discrete from y, and vice-versa.

Another key distinction between our theory and Clarke's concerns the connection between a region and its complement. In the new theory, $\forall x[\mathsf{EC}(x, \mathsf{compl}(x))]$ holds; that is, regions are connected with their complements — which seems a very intuitive result — while in Clarke, a region is disconnected from its complement: $\forall x[\mathsf{DC}(x, \mathsf{compl}(x))]$.

Some further theorems expressing other interesting and important properties of RCC can be found in (Randell, Cui and Cohn 1992) as can a discussion about how to introduce atomic regions into RCC. In the calculus as presented here, they are, of course, excluded because every region has a non-tangential proper part.

4. Expressing Topological Shape in Terms of C

So far, we have principally concentrated on binary predicates relating pairs of regions. Of course, there are also properties of a single region we would like to express, all of which, in some sense at least, characterize the *shape* of the region. Although

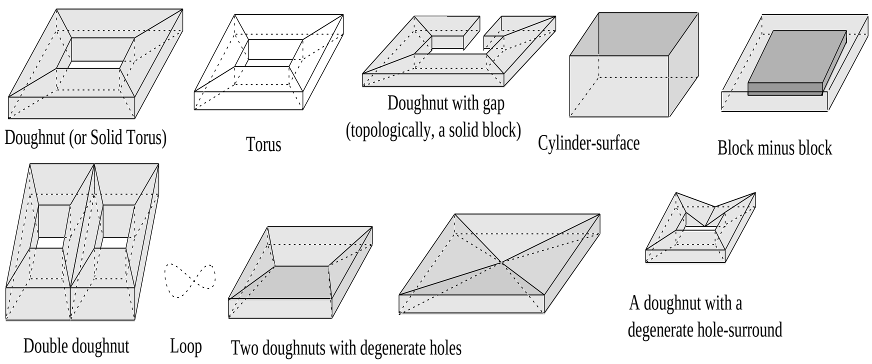

Figure 5. It is possible to distinguish all these shapes using $\mathsf{C}(x, y)$ alone.

we have only developed topological notions there is still quite a bit that can be said about the topological shape of a region. For example we have already introduced the predicate $\mathsf{CON}(x)$ which expresses whether a region is one-piece or not. We can do much more than this however, as (Gotts 1994a, Gotts 1994b, Gotts et al. 1996, Gotts 1996c) demonstrates. The task set there is to be able to distinguish a 'doughnut' (a solid, one-piece region with a single hole). It is shown how (given certain assumptions about the universe of discourse and the kinds of regions inhabiting it) all the shapes depicted in Fig.5 can be distinguished. Here we just give a brief idea of how the task is accomplished, as it also shows some of the range of predicates that can be further defined using C alone (and thus could form the basis of RCCn (for some $n > 8$)).[13]

Gotts defines several classes of predicate describing fundamental aspects of the topology of regions. The *separation-number* (SEPNUM) of a region is the maximum number of mutually disconnected parts it can be divided into. The *finger-connectivity* (FCON) of a CON region is defined[14] in terms of its possible *dissections*, Fig.6 illustrates three different finger connectivities. Making use of an easily definable predicate $\mathsf{MAX\_P}(x, y)$, asserting that x is a maximal one-piece part of y, FCON can be defined. Gotts goes on to define a predicate $\mathsf{SBNUM}(x, y, n)$ to count the number of separate boundaries two regions have in common. Using these definitions a doughnut can be defined as a region with finger connectivity of 2 and a single boundary with its own complement.[15]

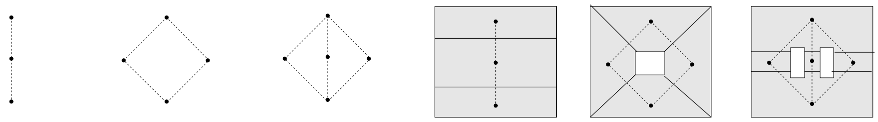

Figure 6. Dissection-graphs and dissections: finger-connectivities 1, 2 and 3

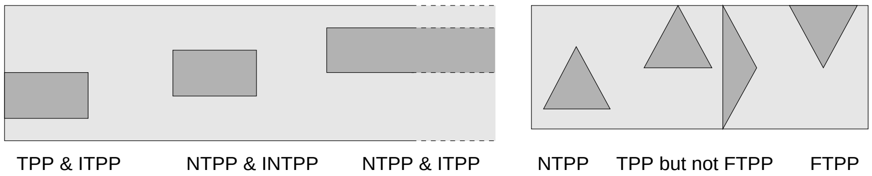

Figure 7. TPPs and ITPPs (left); TPPs and FTPPs (right)

Some of the initial assumptions made by Gotts can be weakened by introducing further defined predicates, which are interesting in their own right. For example, it is possible to define the notion of *intrinsic* TPP, which we term ITPP. Intuitively, x is an ITPP of y iff it is a PP of y that is not surrounded on all sides by the rest of y. Formula (11) defines $\mathsf{TPP}(x, y)$ *extrinsically* since it makes reference to a third region, z, which is predicated to be EC to both x and y. The definition of ITPP avoids this third region. This has the result that u can have an ITPP but it cannot have a TPP: if u is 3-dimensional Euclidean space, any region of infinite diameter is an ITPP of u. ITPP is itself defined in terms of another predicates: $\mathsf{FTPP}(x, y)$ asserts that x is a *firmly* tangential proper part of y (not just point-tangential), which in turn is defined using finger connectivity. These predicates are illustrated in Fig.7.

Fig.8 illustrates another range of topological distinctions between CON regions that can be made (under certain assumptions) using C. A region, if it is connected, may or may not also be interior-connected (INCON), meaning that the interior of the region is all one piece. It is relatively easy to express this property (or its converse) in RCC terms. However, $\mathsf{INCON}(r)$ does not rule out all regions with anomalous boundaries, and in particular does not exclude the region at the right of Fig.8, nor any of the final three cases illustrated in Fig.5, which do have one-piece interiors, but which nevertheless have boundaries which are *not* (respectively) simple curves or surfaces, having 'anomalies' in the form of points which do not have line-like (or disc-like) neighbourhoods within the boundary. (A region in which every boundary-point has such a neighbourhood is called *locally Euclidean*.)

It appears possible within RCC (Gotts 1994b), using the intrinsic ITPP and a similarly intrinsic INTPP, to define a predicate (WCON) that will rule out the INCON but anomalous cases of Fig.8, but it is by no means straightforward,[16] and it is not demonstrated conclusively in (Gotts 1994b) that the definitions do what

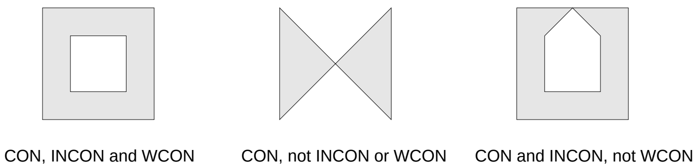

Figure 8. Types of CON Region

is intended. One source of the difficulties arising is the fact that within RCC, since all regions in a particular model of the axioms are of the same dimensionality as u, assuming u itself to be of uniform dimensionality (this follows from the fact that all regions have an NTPP), there is no way to refer directly to the boundary of a region or to the dimensionality of the shared boundary of two EC regions, or to any relations between entities of different dimensionalities.[17] The distinction between intrinsic and extrinsic topological properties, which is found in conventional mathematical topology, is of considerable interest. It demonstrates that at least in some cases, the distinction between properties inherent in an entity and those dependent on its relation with its environment is a real and fundamental one.

5. Increasing Expressive Power: the *Convex Hull* Primitive

Although they are of fundamental importance, it is evident that, for many purposes, purely topological relations are not sufficient to express all significant qualitative spatial properties. The abstraction of topology treats any 2D region bounded by a single continuous and non-self-intersecting curve as equivalent to a disc. Thus, for example, an oval region is topologically equivalent to a long thin wiggly region; but it is clear that, in geography for example, distinguishing between such 'discs' is essential for many useful classification tasks.[18] However it would clearly be very desirable to create more expressive languages for qualitative spatial reasoning but which still fall short of a fully metric descriptive language. An additional primitive (or primitives) clearly needs to be introduced since C is not sufficiently expressive to make such non-topological distinctions. There are many possibilities for the choice of such primitive.

We introduce the notation $\mathrm{RC}\{p_i\}$ to refer to a Region Calculus based on the primitives p_i. Thus the simple RCC theory with just the C relation is denoted $\mathrm{RC}\{\mathsf{C}\}$. We use $\mathcal{B}$ to refer to all the (quasi-)Boolean functions, so that the theory with C plus the Booleans is $\mathrm{RC}\{\mathsf{C}, \mathcal{B}\}$. This notation allows easy reference to further extensions of the the theory with additional primitives, or indeed to replace C with an alternative primitive (for example C can be defined in terms of the more powerful INCH primitive of (Gotts 1996b)).

Apart from C and the Booleans, the primitive to which we have given most attention is conv, a one-place function such that $\mathsf{conv}(x)$ denotes the the *convex hull* of region x. This is the smallest convex region of which x is a part. In Fig.9, the dashed line bounds the convex hull of an imaginary island, 'Concavia'. A region is convex if it is equal to its own convex hull, so a convexity predicate is easily defined by

$$\mathsf{CONV}(x) \equiv_{def} \mathsf{EQ}(x, \mathsf{conv}(x)) \tag{19}$$

The notion of convex hull can itself be defined mathematically in terms of points and lines as the region resulting from including every point on every line joining any two points in the region.

If the properties of convexity are to be adequately captured by inference within the language $\mathrm{RC}\{\mathsf{C}, \mathcal{B}, \mathsf{conv}\}$, we need to specify the logical properties of the new

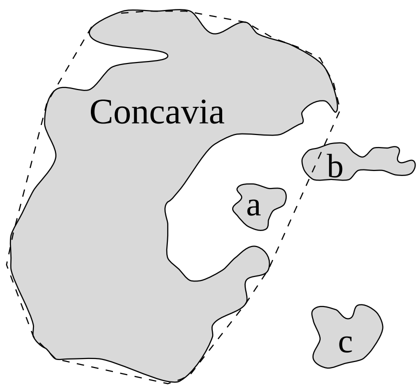

Figure 9. The convex hull of an island

function, $\mathsf{conv}(x)$. Since we have no notion of straight lines, the mathematical definition is not directly applicable. Some important properties of conv can be axiomatized as follows:[19]

$$\forall x[\mathsf{conv}(\mathsf{conv}(x)) = \mathsf{conv}(x)] \tag{20}$$
$$\forall x[\mathsf{TP}(x, \mathsf{conv}(x))] \tag{21}$$
$$\forall x \forall y[\mathsf{P}(x, y) \;\rightarrow\; \mathsf{P}(\mathsf{conv}(x), \mathsf{conv}(y))] \tag{22}$$
$$\forall x \forall y[\mathsf{P}(\mathsf{conv}(x) + \mathsf{conv}(y),\, \mathsf{conv}(x + y))] \tag{23}$$
$$\forall x \forall y[\mathsf{conv}(x) = \mathsf{conv}(y) \;\rightarrow\; \mathsf{C}(x, y)] \tag{24}$$
$$\forall x \forall y[\mathsf{CONV}(\mathsf{conv}(x) * \mathsf{conv}(y))] \tag{25}$$
$$\forall x \forall y[\mathsf{DC}(x, y) \rightarrow \neg\mathsf{CONV}(x + y)] \tag{26}$$

Further discussion of the properties of conv can be found in (Cohn, Bennett, Gooday and Gotts 1997). A complete axiomatisation of convexity is the goal of ongoing research.[20]

5.1. *Containment Relationships and 'Insides'*

Given our new primitive of the convex hull, we can now start defining some new relations that exploit this function symbol. Perhaps the most obvious and useful distinction to make is to distinguish when one region is inside another, that is to say part of its convex hull but not overlapping the region itself. This notion is easy to define. We introduce three new predicates to test for a region being inside another (INSIDE), partly inside another (P-INSIDE) and outside another (OUTSIDE):

$$\mathsf{INSIDE}(x, y) \;\equiv_{def}\; \mathsf{DR}(x, y) \wedge \mathsf{P}(x, \mathsf{conv}(y)) \tag{27}$$
$$\mathsf{P\text{-}INSIDE}(x, y) \;\equiv_{def}\; \mathsf{DR}(x, y) \wedge \mathsf{PO}(x, \mathsf{conv}(y)) \tag{28}$$
$$\mathsf{OUTSIDE}(x, y) \;\equiv_{def}\; \mathsf{DR}(x, \mathsf{conv}(y)) \tag{29}$$

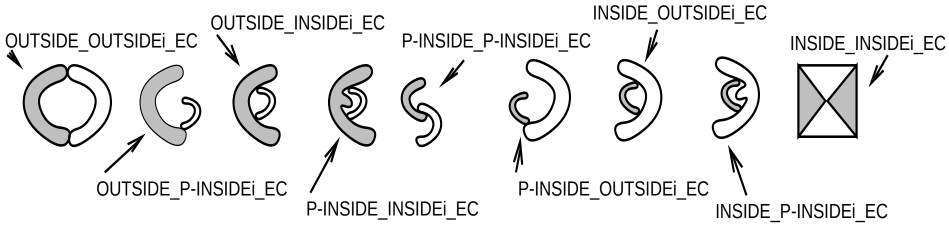

Figure 10. Nine of the new JEPD relations exploiting conv.

Each of these relations is asymmetric so they have inverses, denoted INSIDEi, P-INSIDEi and OUTSIDEi. In terms of the defined relations, the relationships between the small islands in fig 9 and the larger island of Concavia can be described by INSIDE(a, concavia), P-INSIDE(b, concavia) and OUTSIDE(c, concavia).

Obviously we have now moved beyond RCC8, but how many JEPD relations do we now have? It turns out that the above definitions naturally give rise to a set of twenty-three JEPD relations, which we call RCC23. The RCC8 relations of DC and EC no longer form part of the JEPD set; they are replaced by seventeen new relations nine of which are illustrated in Fig.10. The other eight are simply the DC versions of the first eight configurations. These seventeen relations can be schematically defined by

$$\alpha\_\beta\_\gamma(x, y) \equiv_{def} \alpha(x, y) \wedge \beta(y, x) \wedge \gamma(x, y) \tag{30}$$

where $\alpha, \beta \in$ {INSIDE, P-INSIDE, OUTSIDE} and $\gamma \in$ {EC, DC}, excepting the case where $\alpha = \beta =$ INSIDE, and $\gamma =$ DC. This case is ruled out by axiom 24 above.[21] If we ignore the distinction between DC and EC, RCC23 collapses into a set of 15 relations (RCC15) — we shall consider this set later in section 6.4.

Fig.11 depicts an example of how these new relations can be used to advantage when describing the movement of one region from outside to inside and then overlapping another. Region x might, for example, be an oil slick and y an island with a bay. The slick enters the mouth of the bay and ends up partly covering the beach. However, labeling the final configuration simply as PO seems a little unsatisfactory: if x were partially overlapping y on the left-hand side, that is, if it were not within

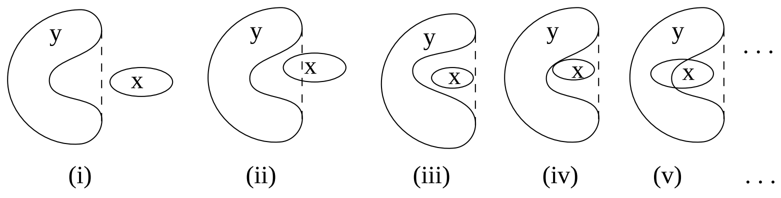

Figure 11. One region moving inside another: (i): OUTSIDE_OUTSIDEi_DC(x, y), (ii): P-INSIDE_OUTSIDEi_DC(x, y), (iii): INSIDE_OUTSIDEi_DC(x, y), (iv): INSIDE_OUTSIDEi_EC(x, y), (v): PO(x, y).

the convex hull of y, it would still be PO. The obvious solution to this is to define PO versions of all the configurations in Fig.10. To do this requires the definitions of INSIDE, P-INSIDE and OUTSIDE to be changed slightly; the details are in (Cohn, Randell and Cui 1995). With this modification there are now thirty-two base relations (the original eight, less DC, EC and PO, plus the EC, DC and PO versions of the allowable combinations of INSIDE, OUTSIDE, P-INSIDE and their inverses).

There are many different reasons why a region may be concave. In particular, regions might be multi-piece, or they may be missing an NTPP rather than having a simple depression in their surface; (Casati and Varzi 1994)[22] is an excellent treatise on the different kinds of holes that might exist.

Fig.12 shows some of the different kinds of non convex regions which can be distinguished using C and conv. The principal distinction we make make is between one region, x, being *geometrically* inside another region, y, and being *topologically* inside y. In the former case, although x is within the convex hull of y it is not 'surrounded' by y — this can be characterised by saying that that there is a one-piece region which overlaps both x and the complement of $\mathsf{conv}(y)$. In the latter case x is completely surrounded by y and no such region exists. This is a very useful distinction to make in many practical situations: for example a frog contained in a jar with its lid on is topologically inside the jar and cannot escape; but, when the lid is removed, it becomes merely geometrically inside and can hop out. The notion of being geometrically inside can be further refined to distinguish those geometrical insides which could contain a liquid — 'containable' insides — and those which are formed by being between components of a multi piece region — 'scattered' insides. Still other possible relationships expressing further refinements could be defined; for example does one region completely fill an inside of another region or does it only partly fill it? Again for many domains this may be a very useful distinction to be able to make easily.

Returning to Fig. 12 we see that in (i) the darker region is geometrically inside the lighter one; in (ii) it is topologically inside; in (iii) it's in the scattered inside. In (iv) a fly in position 3 would be in the containable inside; in position 2 it would be topologically inside (encased in the stem during the manufacturing process!); in position 4 it would be in the 'tunnel inside' (in the 'handle'), while in position 1

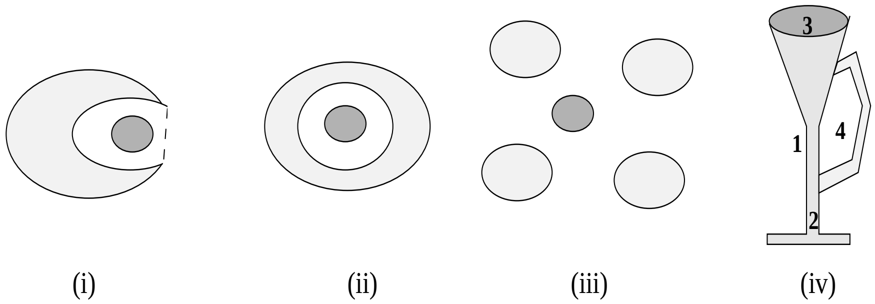

Figure 12. Different kinds of inside that can be distinguished by RCC.

it would be inside the glass in a very weak sense: part of the convex hull but not any of the more specialized senses. (Cohn et al. 1995) provides definitions which distinguish all these kinds of inside and more.

5.2. *Further Properties Definable with* conv

In this section so far, we have concentrated on defining relationships between two regions that exploit the conv primitive. Of course, to a certain extent, these give rise to complementary techniques to describe the shape of one particular region. For example, if one region is topologically inside another, then the second region must have an interior void. In (Cohn 1995) we focus explicitly on defining predicates which characterize the shape of a single two-dimensional region. Techniques are developed, using C and conv alone, which can, for example, distinguish all the different shapes in Fig.13. The principal idea is to distinguish the concavities of a region (which are the maximal one-piece well-connected parts of its inside) and then define predicates that are true of particular configurations of the concavities. Adjacent and non adjacent concavities can easily be distinguished for example. A particularly interesting idea (which turns out to have been long known about in the vision community (Sklansky 1972)) is to apply the technique recursively: if a concavity is itself concave, then one describes the shape of its insides (this is how the first two shapes in Fig.13 are distinguished). Fig.14 illustrates this idea. It is also possible to define when a region is a triangle using C and conv and thus when a region is an arbitrary polygon.

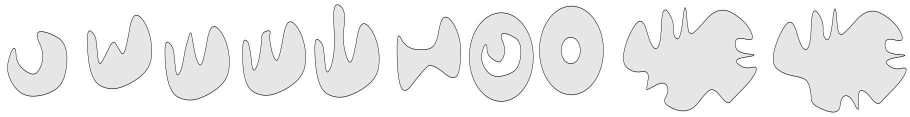

Figure 13. Qualitative predicates can be defined to distinguish all these shapes.

As we have seen, a surprisingly rich and complex and expressive ontology for describing qualitative spatial relationships can be logically defined from just two primitives. It would be easy to define a set of well over a hundred JEPD base

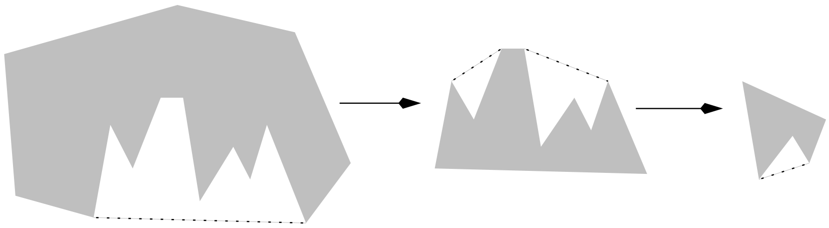

Figure 14. Finer shape descriptions can be obtained by recursively describing the shapes of the insides of a region.

relations from the above predicates described above and many more could easily be defined such as JUST_INSIDE and JUST_OUTSIDE. In fact, (Davis, Gotts and Cohn 1997) shows that the constraint language of RCC-8 together with a monadic CONV(x) predicate (easily definable given conv(x)) yields a decision procedure to distinguish (in 2D) any two regions which are not related by an affine transformation. The question arises: when to stop? In (Cohn et al. 1995) we propose some criteria based on computational and predictive properties of the representation, but ultimately it must be a domain specific question: certain distinctions will only be useful for certain domains, but for these domains they may be crucial.[23] As noted in the introduction, the significance of qualitative distinctions depends largely on their relevance to the behaviour being modeled.

6. Reasoning with the RCC Calculus

So far we have not discussed reasoning with the calculus at all. Of course, since it is expressed in first-order predicate calculus, a wide range of theorem provers are available and indeed we have used these (for example to check the theorems expressed[24] in the lattice of Fig.4 and those in section 3.2 above). However, general 1st-order theorem proving is too inefficient to be useful for most purposes. For certain specific 1st-order theories, special purpose decision procedures can be constructed; but, by reformulating the results of (Grzegorczyk 1951), we can show that the full RCC theory must be undecidable (Gotts 1996d). Nevertheless, as we shall see in the remainder of this section, it is possible to formulate decidable representations, whose vocabulary includes quite expressive sub-languages of RCC.

6.1. Composition Tables

In his temporal calculus Allen introduced the idea of a transitivity table, which we term a *composition table* (following (Freksa 1992)). Given a fixed vocabulary of relations, $\{R_i\}$ (normally this will constitute a JEPD set), such a table enables one to answer the following question by simple lookup: given $R_1(x, y)$ and $R_2(y, z)$, what are the possible relations (from the set $\{R_i\}$) that can hold between x and z?[25] This kind of computation is frequently very useful — for example, one can check the integrity of a database of atomic assertions (involving relations in some set for which we have a composition table) by testing whether every three relations are consistent with the table. We call this 'triangle checking'.[26] Fig.2 gives the composition table for RCC8.[27] Where there are multiple entries this means that a disjunction of relations are possible. We have verified this table by showing that each disjunction has a possible model in the intended interpretation and by proving each entry is a theorem of the form $\forall x \forall y \forall z[(R_1(x, y) \wedge R_2(y, z)) \rightarrow R_3(x, z)]$.

Because of the extreme difficulty of general 1st-order reasoning, even verifying the composition table for RCC8 using the Otter theorem prover (McCune 1990) was a hard task requiring introducing various lemmas by hand (Randell, Cohn and Cui 1992). It became clear that this approach would not scale up to RCC23 or larger sets of relations and that more efficient reasoning techniques would be required. Initially,

Table 2. Composition table for RCC8

$R1(a,b)$ \ $R2(b,c)$	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	no.info	DR,PO,PP	DR,PO,PP	DR,PO,PP	DR,PO,PP	DC	DC	DC
EC	DR,PO,PPi	DR,PO TPP,TPi	DR,PO,PP	EC,PO,PP	PO,PP	DR	DC	EC
PO	DR,PO,PPi	DR,PO,PPi	no.info	PO,PP	PO,PP	DR,PO,PPi	DR,PO PPi	PO
TPP	DC	DR	DR,PO,PP	PP	NTPP	DR,PO TPP,TPi	DR,PO PPi	TPP
NTPP	DC	DC	DR,PO,PP	NTPP	NTPP	DR,PO,PP	no.info	NTPP
TPPi	DR,PO,PPi	EC,PO,PPi	PO,PPi	PO,TPP,TPi	PO,PP	PPi	NTPPi	TPPi
NTPPi	DR,PO,PPi	PO,PPi	PO,PPi	PO,PPi	O	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

we experimented with a model-building program based on bitmap representations of possible spatial situations but this was only partly successful and was still fairly computationally intensive. We needed a more tractable logical representation of our theory.

6.2. A Zero-Order Encoding of RCC

The idea we pursued was to move from a first-order representation to a zero-order logic, which then provided a decision procedure (Bennett 1994b). Here we summarize this approach to RCC. Zero-order logic is traditionally known as propositional logic; however, this is an inappropriate name for our purposes. We will not interpret the non-logical symbols as propositions (having truth values) but rather as symbols having sets as their values. If these sets are sets of spatial entities (points or atomic regions for example), then the non-logical symbols denote spatial regions, and logical connectives correspond to certain functions from regions to regions. Suppose we then assert that some formula denotes the universal region: this means that the regions denoted by the 'propositional' constants occurring in the formula must stand in some particular (spatial) relationship, determined by the logical structure of the formula. The formula can thus be used to represent that relationship.

We say that such a spatial interpretation is *faithful* to a propositional logic if entailment among formulas in the zero-order representation mirrors entailment among the corresponding spatial relations. Such an interpretation can be used to reason about a certain class of spatial relationships. For example, the classical formula $A \rightarrow B$ can be used to represent the relation $\mathsf{P}(a,b)$ and the entailment $A \rightarrow B,\ B \rightarrow C\ \models\ A \rightarrow C$ reflects the fact that $\mathsf{P}(a,b),\ \mathsf{P}(b,c)\ \models\ \mathsf{P}(a,c)$.

It turns out that classical zero-order logic is not sufficiently expressive to encode the RCC8 relations. Happily, this can be achieved in *intuitionistic* zero-order logic (an introduction can be found in (Nerode 1990)), which we name $\mathcal{I}_0$. In fact, the idea of a topological interpretation of this logic was first introduced by Tarski (1938), who gave a mapping from intuitionistic formulae to open sets in a topological space, such that all intuitionistic theorems are mapped to the universal set, $\mathcal{U}$. Intuitionistic logic is weaker than classical logic in that certain classical theorems

Table 3. RCC relations defined in $\mathcal{I}_0^+$

Relation	*Model Constraint*	*Entailment Constraints*
$\mathsf{DC}(X,Y)$	$\sim X \vee \sim Y$	$\sim X,\ \sim Y$
$\mathsf{EC}(X,Y)$	$\sim(X \wedge Y)$	$\sim X \vee \sim Y, \sim X,\ \sim Y$
$\mathsf{PO}(X,Y)$	—	$\sim(X \wedge Y),\ X \Rightarrow Y,\ Y \Rightarrow X, \sim X, \sim Y$
$\mathsf{TPP}(X,Y)$	$X \Rightarrow Y$	$\sim X \vee Y,\ Y \Rightarrow X, \sim X, \sim Y$
$\mathsf{TPPi}(X,Y)$	$Y \Rightarrow X$	$\sim Y \vee X,\ X \Rightarrow Y, \sim X, \sim Y$
$\mathsf{NTPP}(X,Y)$	$\sim X \vee Y$	$Y \Rightarrow X, \sim X, \sim Y$
$\mathsf{NTPPi}(X,Y)$	$\sim Y \vee X$	$X \Rightarrow Y, \sim X, \sim Y$
$\mathsf{EQ}(X,Y)$	$X \Leftrightarrow Y$	$\sim X, \sim Y$

do not hold. In particular, the law of the excluded middle, $p \vee \sim p$ does not hold for all propositions. Under Tarski's interpretation, p would denote some open set, P, and $\sim p$, the *interior* of the complement of P. Disjunction is identified with set-theoretic union, so that $p \vee \sim p$ denotes the set $P \cup interior(\overline{P})$. Because this set does not include the *boundary* of P, it is not equal to the universe.

This semantics associates each formula of $\mathcal{I}_0$ with a term involving constants denoting open sets, the Boolean set-theoretic operators and the interior operator. We call this a *set-term* . If regions are considered as open sets, a formula can be used to represent that spatial relation which holds between regions just in case the corresponding set-term has the value $\mathcal{U}$. This is a faithful interpretation of $\mathcal{I}_0$, which means that a standard theorem prover for $\mathcal{I}_0$ can be used to reason about spatial relations represented in this way. However, $\mathcal{I}_0$ is still not quite sufficient by itself to distinguish all the RCC8 relations. It turns out that we need not only conditions expressible by asserting that some set-term equals $\mathcal{U}$ but also conditions that require us to assert that some set-term does not equal $\mathcal{U}$. For example, although the part relation $\mathsf{P}(a,b)$ is straightforwardly represented by the relation $A \Rightarrow B$, representing the proper part relation $\mathsf{PP}(a,b)$ requires us, in addition, to ensure that the relation $\mathsf{P}(b,a)$ does not hold. It turns out that this limitation can be overcome with a simple extension of $\mathcal{I}_0$, which we term $\mathcal{I}_0^+$, together with an appropriate meta-level reasoning algorithm. Expressions of $\mathcal{I}_0^+$ are pairs of sets of $\mathcal{I}_0$ formulas, $\langle \mathcal{M}, \mathcal{E} \rangle$. One set represents (positive) *model constraints*; the other (negative) *entailment constraints*. For example, $\mathsf{PP}(p,q)$ may be represented as $\langle \{p \rightarrow q\}, \{q \rightarrow p\} \rangle$.[28] The $\mathcal{I}_0^+$ encoding enables all the RCC8 relations to be defined as shown in table 3.

Bennett (1994b) explains the $\mathcal{I}_0^+$ representation in detail and proves the correctness of the following algorithm to determine the consistency of sets of spatial relations represented in $\mathcal{I}_0^+$:

1. For each relation $R_i(\alpha_i, \beta_i)$ in the situation description find the corresponding propositional representation $\langle \mathcal{M}_i, \mathcal{E}_i \rangle$.
2. Construct the overall $\mathcal{I}_0^+$representation $\langle \bigcup_i \mathcal{M}_i,\ \bigcup_i \mathcal{E}_i \rangle$.
3. For each formula $F \in \bigcup_i \mathcal{E}_i$ use an intuitionistic theorem prover to determine whether the entailment[29] $\bigcup_i \mathcal{M}_i \models F$ holds.
4. If any of the entailments determined in the last step does hold, then the situation is impossible.

A slightly more complicated algorithm will test entailment rather than consistency. (Bennett 1994b) also presents a method of capturing certain properties of the $\mathsf{conv}(x)$ function in the zero-order representation.

It is worth pointing out that Bennett's $\mathcal{I}_0^+$ representation gives us a true spatial logic rather than simply a logical theory of space: the logical constants ($\wedge, \vee, \Rightarrow$ etc.) all have a spatial interpretation. We have also investigated other possible spatial logics, in particular modal ones where the necessity operator is interpreted as an interior operator (Bennett 1995). Another investigation of the use of modal logics for RCC, interpreting C as the accessibility operator can be found in (Cohn 1993).

6.3. The Complexity of Reasoning with RCC

We have noted that, since first-order logic is undecidable, the original formulation of RCC as a first-order theory does not provide us with an effective inference mechanism for the language. However, many highly expressive sub-languages of RCC can be specified as *constraint languages* consisting of sets of properties and relations definable in the RCC theory or one of its extensions. Such a constraint language provides a fixed vocabulary of spatial predicates which may be chosen for its computational properties and/or its relevance to a particular domain. The complexity of reasoning with various spatial constraint languages is the subject of much current research.[30]

The RCC8 relations constitute a constraint language which is of fundamental importance. Nebel (1995a) has shown that Bennett's $\mathcal{I}_0^+$ reasoning algorithm, when applied to instances of the RCC8 relations, has polynomial complexity in the number of instances.[31] Some other complexity results for reasoning with the RCC8 relations are given in (Grigni, Papadias and Papadimitriou 1995). This paper is concerned with relational consistency and also with *realisability* of a set of relations by a set of simply-connected planar regions. Drawing on results of Kratochvíl (1991) about the recognition of realisable string graphs Grigni et al. (1995) conclude that testing realisability is NP-hard. Another important result, that the constraint language of RCC-8 plus a predicate $\mathsf{CONV}(x)$ is decidable but "at least as hard as determining whether a set of comparable size of algebraic constraints over the real numbers is consistent" has been demonstrated in (Davis et al. 1997).

6.4. Reasoning about Continuous Change

So far we have concerned ourselves only with expressing the static properties of space rather than with developing a calculus for expressing how configurations of spatial regions evolve over times. However, such dynamic reasoning is clearly very important in many situations. In many domains, an assumption is made that change is *continuous*. The QR community has exploited this notion repeatedly (see, for example, (Weld and De Kleer 1990)). In the context of qualitative *spatial* reasoning, assuming continuity means assuming that shape deformations are continuous

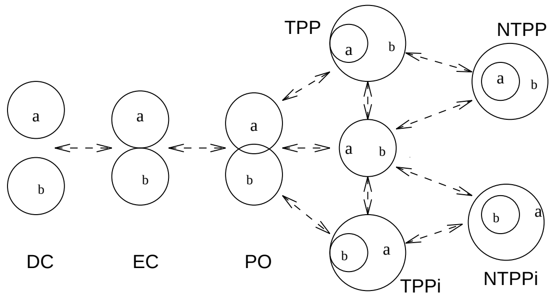

Figure 15. The continuity network for RCC8

in addition to assuming that movement is continuous. Fig.15 indicates possible state transitions among the RCC8 relations assuming continuity and fig. 16, which we call a *continuity network* indicates continuous transitions among the RCC15 relations.[32] Subgraphs of a continuity network turn out to coincide the notion of *conceptual neighbourhoods* introduced by Freksa (1992). Galton, in his chapter in (Stock 1997) and in (Galton 1995b) has made a thorough analysis of continuity as it applies to RCC8. (Egenhofer and Al-Taha 1992) builds similar (though not identical) structures for his calculus (there are fewer links in general) using a notion called *closest topological distance.* In section 7.3 we discuss how these kinds of structures can be used to build qualitative spatial simulators.

We and others have noticed an interesting relationship between composition tables and continuity networks. For a variety of calculi, every entry in a composition table forms a connected subgraph of the continuity network.[33] For example, an entry that included DC and PO would also have to include EC. Freksa (1992) exploited this to generate a compact composition table for Allen's system. Freksa's

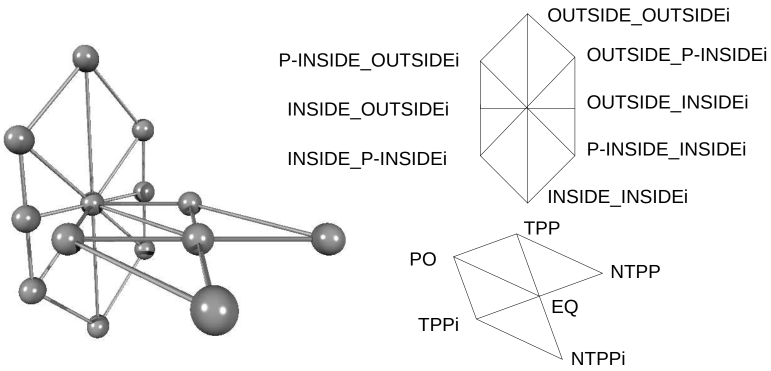

Figure 16. The continuity network for RCC15. The unlabelled central node of the vertical plane is P-INSIDE_P-INSIDEi. PO has links to this and every other node in the vertical plane.

reduced table gives compositions for sets of relations (conceptual neighbourhoods, in fact) rather than single relations. We have explored this approach in the context of Allen's calculus and the RCC system (Cohn, Gooday and Bennett 1994). By slightly relaxing Freksa's conditions for choosing sets of relations upon which to base the tables, we managed to find compact representations for certain sets of RCC relations; for example a 6×6 solution (44 per cent reduction in table size) for RCC8 and 8×8 solution for RCC15 (75 per cent reduction in table size). We also investigated the construction of neighbourhood graphs from information in composition tables and have had some success in this venture using a constraint-based approach.

7. Some Applications of RCC

The main focus of our work has been theoretical: to design logical calculi for qualitative spatial reasoning. However, we have also worked on applying RCC to some specific domains. In this section we describe applications of the calculus to: evaluating queries within a geographical information system (GIS); describing geographical features; qualitative simulation of physical changes; and specifying the semantics of a visual programming language.[34]

7.1. RCC in Geographical Information Systems

An obvious application of RCC, to which we have given attention (Cohn, Gotts, Randell, Cui, Bennett and Gooday 1997, Bennett 1996a), is geographical information systems (GIS). In fact, a parallel development of a system very similar to RCC8 has taken place within this field (Egenhofer and Franzosa 1991, Egenhofer 1991, Egenhofer, Clementini and Di Felice 1994, Egenhofer 1994, Egenhofer and Franzosa 1995, Clementini, Di Felice and Oosterom 1994, Haarslev and Möller 1997) but firmly based on a point-set theoretic approach rather than our logic of regions approach.

The topological reasoning algorithm based on encoding RCC relations in $\mathcal{I}_0^+$ (described in section 6.2 has been implemented as part of a larger 'spatial AI' system being developed as part of EPSRC project GR/K65041 on 'Logical Theories and Decision Procedures for Reasoning about Physical Systems'. The current system contains a database of geographical information in the form of geometrical polygon data and also contains qualitative data in the form of topological relations between named regions. Some of these named regions are identified directly with polygons in the geometrical database, whereas for others the geometry is not precisely known but only constrained by the qualitative topological relations. The topological relationships determined by the quantitative geometrical data can also be rapidly computed and accessed by the topological reasoning mechanism, allowing queries to be addressed to the combined qualitative and quantitative database. This capability is (as far as we know) not available in any other system. Work is also underway to demonstrate the use of topological reasoning in the control of artificial agents operating in a virtual world constituted by geographical data.

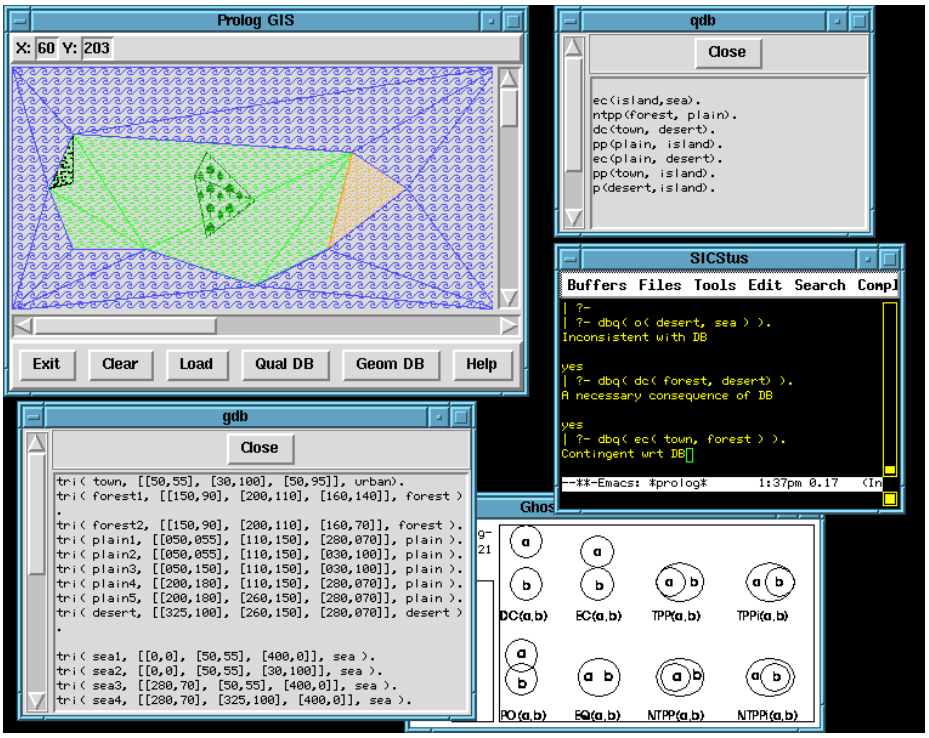

Figure 17. Our current GIS prototype

Fig.17 shows a screen-dump of the current prototype system. Most of the code is written in (SICStus) Prolog but a Tcl/Tk sub-process is used to create the GUI. The window at the top left shows a simple cartographical display, whose geometry is determined by a database giving the coordinates and terrain type of a number of triangular regions. This data is shown in the bottom left window. The top right window presents a database of qualitative relations between regions. In the middle on the right is the Prolog top-level query window. All functions of the system can be accessed by typing commands and queries at the Prolog prompt (although common operations are more conveniently accessed via the GUI). The figure shows the Prolog interpreter being used for querying the qualitative database. Such queries are answered by means of Bennett's consistency checking algorithm which will determine whether a relation given as a query is consistent with, inconsistent with or a necessary consequence of the database. (The bottom right window is one of a number of information screens which can be displayed via the system's 'help' function.)

In developing our GIS prototype, we have become very much aware of the importance of integrating qualitative and quantitative spatial information, if useful functionality is to be obtained. This is an area upon which we intend to focus in future research.

7.2. *Characterisation of Geographical Features*

Figure 9 (section 5 above) illustrates the use of the convex-hull concept in distinguishing among geographical relationships. We see a large island and three smaller islands — a, b and c. Each island is disconnected with the others, so topologically the relation between any two islands is the same. However, if we consider the relation between each of the three small islands and the convex-hull of the large island, we see that: a is part of this convex-hull; b overlaps the convex-hull; and, c is disconnected from the convex hull. Moreover, this qualitative difference is significant from a geo-physical point of view. The tidal and weather conditions affecting a, lying within a bay of the larger island, are likely to be different from those affecting c out in the open sea. Given a differentiation of regions into land and sea, the concept of a 'bay' region can be defined quite straightforwardly in terms of convex-hull together with purely topological concepts. A bay is a maximal[35] one-piece sea region which is part of the convex-hull of a land region.

There is some scope for argument as to whether this really captures the concept of 'bay'. For instance if we have a long coastline whose curvature over its whole length is concave, then this always creates a single 'bay' according to our definition and no smaller concavity on this coastline is counted as forming a bay. One might contend that this rules out many features that ought to be classified as bays. But this kind of problem does not count against the value of the approach. It shows that either the concept of 'bay' is ambiguous or it requires a more complex definition along the same lines or it cannot adequately be defined purely in terms of topology, convexity and the land/sea distinction. In the first case the analysis serves to disambiguate what is meant by a 'bay'. The second possibility motivates further investigation of the classification of different kinds of concavity in terms of convex-hull (as we discussed in sections 5.1 and 5.2). Similarly the third case motivates inquiry into what other primitive concepts may be needed to define geographical features.

To further illustrate the power of RCC we consider how the spatial extension of the geographical feature known as an 'ox-bow lake' might be characterised by means of a predicate definition in the theory RC$\{\mathsf{C}, \mathcal{B}, \mathsf{conv}\}$. Fig. 18 depicts the formation of a typical ox-bow lake. In the first picture, a river is meandering across its flood plain; in the second picture erosion has caused the river to break through the meander; in time silting on the slower flowing original segment causes the separation of the original meander creating an ox-bow lake. Thus an ox-bow is typically of a crescent like shape with the mouth of the crescent towards the river.

First we define a crescent using the concept of geometric inside. Recall that a region x is geometrically inside region y if x is part of $\mathsf{conv}(x)$ but neither overlaps y nor is completely surrounded by y (see Fig.12). We say that the maximal region x satisfying these conditions is *the* geometrical inside of y and define a function $\mathsf{geoinside}(x)$ to map regions to their geometric insides (in many cases $\mathsf{geoinside}(x)$ will, of course, be the null region). Consider the following definition: x is crescent

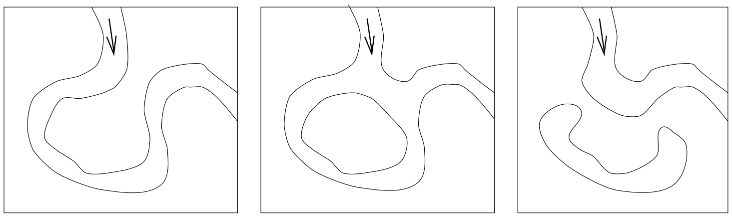

Figure 18. The formation of an ox-bow lake.

shaped iff it is concave and the sum of x and $\mathsf{geoinside}(x)$ is convex (i.e. the whole inside of x is its geometric inside) and its inside is in one piece. This essentially describes a single convex region but with a single "bite" taken out of it. Since we will want to refer to the "mouth" of a crescent below, we make Crescent dyadic so that $\mathsf{Crescent}(m, x)$ means that m is the mouth of a crescent, x.

$$\begin{aligned}\mathsf{Crescent}(m, x) \ \equiv_{def}\ & \neg\mathsf{CONV}(x) \wedge \mathsf{CONV}(x + \mathsf{geoinside}(x)) \wedge \\ & \mathsf{CON}(\mathsf{geoinside}(x)) \wedge m = \mathsf{geoinside}(x)\end{aligned} \tag{31}$$

This is only a first attempt at defining the shape of an ox-bow lake. It is inadequate on at least two counts: firstly there might be more than one concavity because x has small local curvatures around its perimeter; secondly because the perimeter x could be completely made up of straight line segments. The first problem is not so easy to fix within the present framework: it requires one to abstract overall shape from minor local boundary changes. This could be achieved either by adding some shape abstraction operator to the language or perhaps by adding a primitive which would allow one to compare the sizes of regions.[36] If one concavity was much larger than any of the other concavities then it might identified as the mouth of the crescent.

It is quite easy to ensure that a region has a completely curved boundary by means of the following defined predicate:

$$\begin{aligned}\mathsf{Curved}(x) \ \equiv_{def}\ & \forall y[(\mathsf{PO}(x, y) \wedge \mathsf{CONV}(y)) \rightarrow \\ & (\neg\mathsf{CONV}(y - x) \vee \neg\mathsf{CONV}(y * x))]\end{aligned} \tag{32}$$

This ensures that for any convex region y partly overlapping x, either the part of y exterior to x or the part interior to y are concave. Hence, x can have no straight lines, because a convex region which partly overlaps by crossing a straight boundary segment would always be divided into two convex parts. This requirement may not be precisely what is wanted because an ox-bow lake might (up to a given precision of measurement) be straight along some segment of its boundary.

Assuming we are happy for the present with defining the shape of an ox-bow lake as a 'Curved Crescent', we now need to ensure that it is appropriately oriented, i.e. with the mouth of the crescent towards a river. The characterisation of a river is itself an interesting problem leading to many further considerations, however, for the purpose of this example we shall assume that a predicate $\mathsf{River}(x)$ has already been defined. We can then achieve our goal quite simply by requiring that a convex region can overlap both the mouth of the lake and the river without overlapping the lake itself.

$$\begin{aligned}\mathsf{OxBowLake}(x) \ \equiv_{def} \ \exists m \exists r \exists c[&\mathsf{Crescent}(m, x) \wedge \mathsf{River}(\mathsf{r}) \wedge \mathsf{CONV}(c) \wedge \\ &\mathsf{DR}(m, x) \wedge \mathsf{O}(c, m) \wedge \mathsf{O}(c, r)]\end{aligned} \tag{33}$$

Our work on characterising geographical features is at an early state. We realise that the analysis given this section leaves many questions unanswered but hope that it illustrates what we believe to be an important potential use of the RCC calculus.

7.3. Qualitative Simulation of Spatial Changes

It is not difficult to build a qualitative spatial simulator based on composition tables and conceptual neighbourhoods as described in (Cui et al. 1992).[37] A state is a conjunction of ground atomic atoms expressed in RCCn. Successor states are generated by forming the set of neighbouring atoms (using the conceptual neighbourhood diagram) for each atom in the state and forming the crossproduct of all these sets. Each successor state can then be checked for logical consistency by 'triangle checking' using the composition table (see section 6.1 above). It is useful to allow the user to specify domain-dependent inter- and intra-state constraints that further filter which next states are indeed allowable. The implementation also allows users to specify 'add' and 'delete' rules to introduce new regions under certain conditions, with specified relationships to existing regions, or to delete specified regions. Fig.19 illustrates two paths from envisionment generated by the program on a model of phago- and exco-cytosis (an amoeba eating a food particle and expelling the waste matter).[38] It should be clear that this approach can also be applied to modelling geographical processes — the regions might correspond to terrain types or hydrological features.

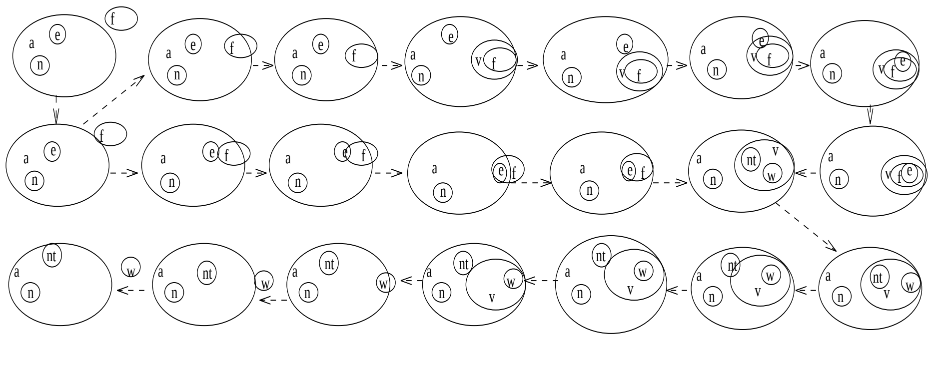

Figure 19. Two paths from the amoeba simulation; the amoeba is denoted 'a', its nucleus 'n', the food particle, 'f', an enzyme, 'e', the vacuole in which the food particle is trapped, 'v', the nutrient formed by digestion, 'nt' and the waste matter, 'w'.

We are now constructing a new qualitative simulation system using Transition Calculus (Gooday and Galton 1996), an event-based nonmonotonic temporal reasoning formalism. This simulator has a much more formal basis than our original qualitative spatial simulator described in section 7.3 above and has already been used to model a simple physical system. RCC continuity networks can be directly represented as event types in Transition Calculus' high-level modeling language making it well-suited to our simulation tasks. We intend to encode various continuity networks and explore a number of simulation problems with the new system.

7.4. Semantics for a Visual Programming Language

Another application we are currently investigating is for the specification of the syntax and semantics of a visual programming language (Gooday and Cohn 1995, Gooday and Cohn 1996b). Visual programming languages are an important new weapon in the software engineer's armory, but while textual languages have benefited from work on providing appropriate mathematical semantics, there has been little work on providing suitable tools for visual languages. One visual language that can be specified almost entirely using topological concepts is Pictorial Janus (Kahn and Saraswat 1990) and indeed RCC turns out to be quite suitable for this task.[39] Fig.20 illustrates some basic Pictorial Janus elements and a program to append two lists.

A constant consists of a closed contour (the shape is irrelevant) containing a number or string (what the constant represents) and a single internal port. The internal port is represented by another closed contour abutting the constant but wholly inside it and acts as a handle for the entire object. Ports cannot themselves contain any elements. Functions are represented by closed contours containing a label and an internal port together with any number of external ports. In this case we have illustrated a list-constructor function, cons, which normally takes two arguments and thus requires two external ports. The final part of the figure shows how the cons function can be used to build up a list.

A Pictorial Janus agent is a closed contour containing rules, a call arrow to another agent contour, or a label. It may have any number of external ports but

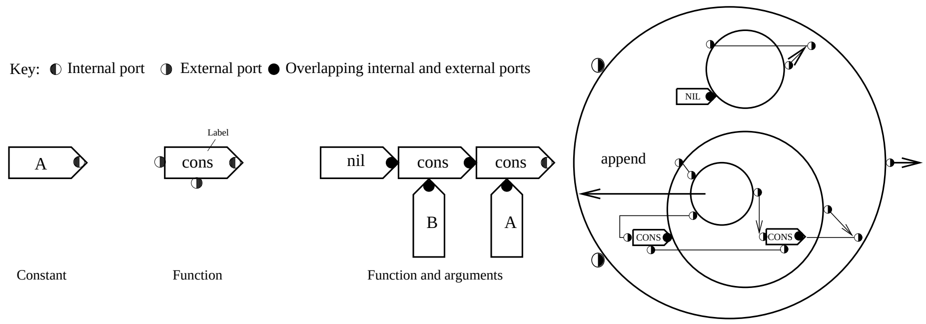

Figure 20. Some basic Pictorial Janus concepts and an append agent (containing two rules).

no internal ports. A rule is defined in exactly the same way as an agent but with the additional requirement that it must be contained within an agent. Agents may communicate via channels: directed curves linking two ports (an arrow is used to indicate directionality). Finally, links are undirected curves joining two ports. There is not space to fully specify Pictorial Janus here, but as a simple example we will give the definitions for internal and external ports that are defined in terms of ports, to show how RCC can be exploited in this domain:

$$\mathsf{Iport}(port, x) \equiv_{def} \mathsf{Port}(port) \wedge \mathsf{TPP}(port, x) \quad (34)$$

$$\mathsf{Eport}(port, x) \equiv_{def} \mathsf{Port}(port) \wedge \mathsf{EC}(port, w) \quad (35)$$

Using RCC we have successfully captured the full syntax of Pictorial Janus and are now working on completing our description of the procedural semantics. It is intended that these RCC descriptions will be used in conjunction with our spatial simulator to model the execution of Pictorial Janus programs.

Another application we have investigated is the application of RCC to help with the problem of integrating two different databases. In this case we are using a spatial metaphor: we think of a database class as a region, and the prototypical members as another region which is always a PP of the complete class.[40] The question addressed in this work is how can we obtain a measure of the reliability of the merge of two data-types? For example, supposing firm A takes over firm B and they merge their employee databases. They may have different definitions of employee. We use a spatial metaphor to develop a ranking to rate the relative goodness of fit in such cases. The final ranking we developed is a refinement of the run of our qualitative simulator. Further details can be found in (Lehmann and Cohn 1994).

8. Spatial Regions with Uncertain Boundaries

Much work in qualitative spatial reasoning is based exclusively on crisp regions and lines. But many domains, particularly GIS, have objects with indeterminate boundaries, such as clouds, urban areas, areas of a certain soil or vegetation type, marshlands, habitats and so on. The question is whether RCC theory as developed so far can be used or extended to model these kinds of entities.[41] In a series of papers (Cohn and Gotts 1996a, Cohn and Gotts 1994b, Cohn and Gotts 1994a, Gotts and Cohn 1995, Cohn and Gotts 1996b) we have tackled this problem[42] from two sides: firstly we have added a further primitive and developed an axiomatisation and a series of definitions to help model such indeterminate spatial entities; secondly, we have applied the *egg-yolk* calculus, mentioned above, to represent such regions.

8.1. A Primitive for Reasoning about Indeterminacy

We need to say at least some of the same sorts of things about vague regions as about crisp ones, with precise boundaries: that one contains another (southern England contains London, even if both are thought of as vague regions), that two overlap (the Sahara desert and West Africa), or that two are disjoint (the Sahara and Gobi deserts). In these cases, the two vague regions represent the space occupied by distinct entities, and we are interested in defining a vague area corresponding to the space occupied by either, by both, or by one but not the other. We may also want to say that one vague region is a crisper version of another. For example, we might have an initial (vague) idea of the extent of a mineral deposit, then receive information reducing the imprecision in our knowledge. Here, the vagueness of the vague region is a matter of our ignorance: the entity concerned actually occupies a fairly well-defined region — though perhaps any entity's limits will be imprecise to some degree. In other cases, vagueness appears intrinsic: consider an informal geographical term like 'southern England'. The uncertainty about whether particular places (north of London but south of Birmingham) are included cannot be resolved definitively: it is a matter of interpretational context. A contrasting example is the region occupied by a cloud of gas from an industrial accident. Here we have two sources of intrinsic vagueness: the concentration of the gas is likely to fall off gradually as we move out of the cloud; and its extent will also vary over time, so any temporal vagueness (for example, if we are asked about the cloud's extent at 'around noon') will result in increased spatial vagueness. In these cases of intrinsic vagueness, there is a degree of arbitrariness about any particular choice of an exact boundary, and often, none is required. But *if* we decide to define a more precise version (either completely precise, or less vague but still imprecise), our choice of version is by no means *wholly* arbitrary: we can distinguish more and less reasonable choices of more precise description. Distinguishing ignorance-based from intrinsic vagueness is important, but many of the same problems of representation and reasoning arise for both.

This then motivates introducing an additional primitive: a binary predicate $X \prec Y$,[43] read as "X is crisper than Y", which is axiomatized to be asymmetric and transitive and hence irreflexive. Various useful predicates can easily be defined in terms of $X \prec Y$. For example, Crisp(X), which is true when no region is crisper than X, MA(X,Y), which is true when X and Y are mutually approximate; that is, they have a common crisping, and $X \prec\!\!\prec Y$ which is true when X is crisper than Y and is itself crisp.

$$\mathsf{Crisp}(X) \;\equiv_{def}\; \neg\exists Y[Y \prec X] \tag{36}$$

$$\mathsf{MA}(X,Y) \;\equiv_{def}\; \exists Z[Z \prec X \wedge Z \prec Y] \tag{37}$$

$$X \prec\!\!\prec Y \;\equiv_{def}\; X \prec Y \wedge \mathsf{Crisp}(X). \tag{38}$$

Further axioms postulate the existence of a complete crisping of any region, and also of alternative ways to crisp and decrisp a region (for if this were not so, then one could hardly claim that indeterminacy existed about the region). Another possible axiom asserts the denseness of crisping: if $X \prec Y$, then there must be another region crisper than Y but less crisp than X. An interesting parallel can be drawn between this theory and the axiom-sets for mereology (theory of part-whole relations) discussed by (Simons 1987); we will return to this below.

The question arises: how many JEPD relations are there between non crisp regions? For the sake of simplicity, we consider a calculus for spatial regions with

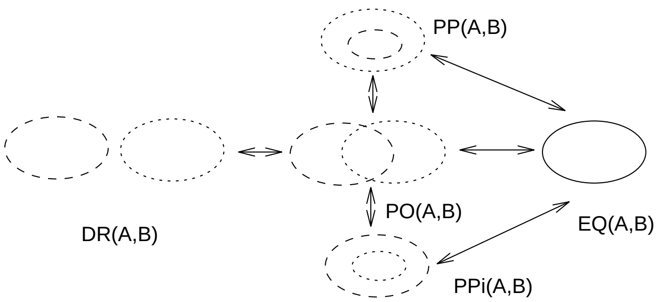

Figure 21. The RCC5 continuity network.

indeterminate boundaries based on the fairly coarse-grained relation set which we call RCC5, consisting of the relations DR, PO, PP, PPi and EQ.[44] Fig.21 depicts the RCC5 relations and their continuity network. Consider two non-crisp regions. Depending on the initial configurations, there may be different possible RCC5 relations between complete crispings of the two regions. We make the assumption that any set of complete crisping relations will be a conceptual neighbourhood (a connected subgraph of the continuity network). Although there are twenty-three such conceptual neighbourhoods for RCC5, (see Fig.21) it is possible to argue that only thirteen[45] of these can form a set representing the possible complete crispings a pair of vague regions. However, in the next section we suggest that more than thirteen distinctions are, in fact, possible.

8.2. The Egg-Yolk Theory

We have already mentioned the egg-yolk theory above when discussing the application of RCC theory to database integration. Fig.22 depicts this representation. The *egg* is the maximal extent of a vague region and the *yolk* is its minimal extent, while the *white* is the area of indeterminacy. Note that since RCC allows non connected regions, so yolks (and indeed eggs themselves) could be multi-piece. Fig.23 shows the forty-six possible relations between two non-crisp regions (assuming that RCC5 calculus is used to relate eggs and yolks and that yolks are never null).[46]

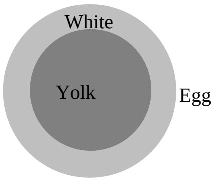

Figure 22. The egg/yolk interpretation

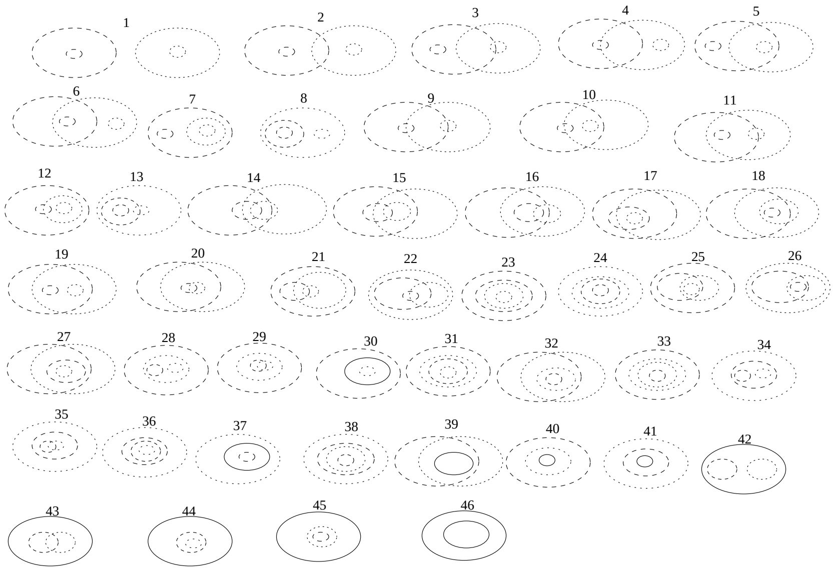

Figure 23. Forty six egg-yolk relations between two eggs

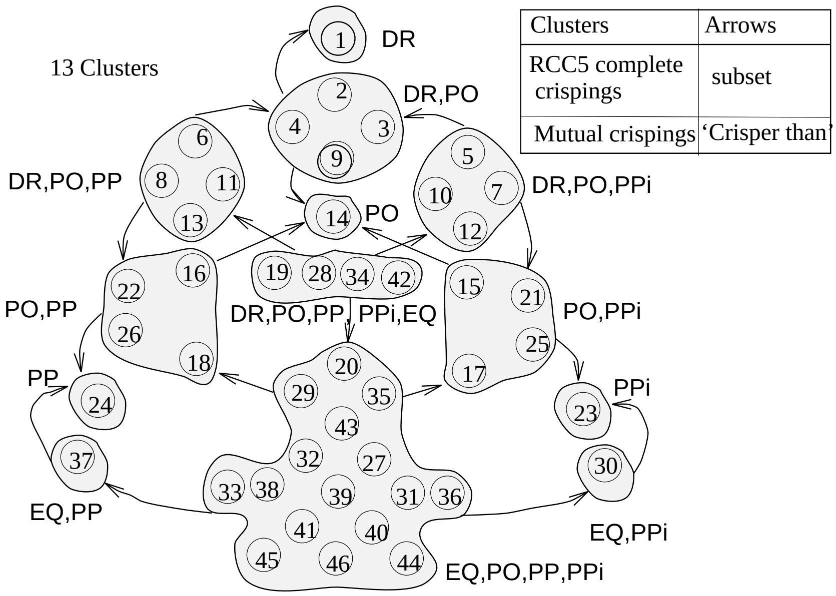

Figure 24. Clustering the forty six relations into thirteen groups.

These forty six relations can be naturally clustered into thirteen groups as shown in Fig.24; this may be achieved either by considering the equivalence classes of configurations that have the same set of relationships between their complete crispings or by grouping together all configurations which may be transformed into each other by crisping.

At first glance, there is an apparent problem with the egg-yolk approach: the most obvious interpretation is that it replaces the precise dichotomy assumed in the basic RCC theory, where space is divided into what is in a region and what is outside a region, by an equally precise trichotomy of yolk, white and outside. This appears contrary to a key intuition about vagueness: that not only is there a doubtful zone around the edges of a vague region but that this zone itself has no precise boundaries. Gotts and Cohn (1995) suggest a way of using the egg-yolk formalism that is consistent with this.

We link the $\mathsf{OCregions}$ of Section 8.1 (and the corresponding theory), with ordered pairs of RCC5 regions, the first of the pair being a part, but not necessarily a proper part, of the second. If it *is* a PP, then the pair is an egg-yolk pair in the sense of (Lehmann and Cohn 1994), and the $\mathsf{OCregion}$ is $\mathsf{NonCrisp}$. If not, the $\mathsf{OCregion}$ is Crisp. We now link the CR predicate of $\mathsf{OCregion}$ theory with the egg-yolk approach. We define a function ey to map an $\mathsf{OCregion}$ to an egg-yolk pair, and two functions eggof and yolkof, to map such egg-yolk pairs to the RCC5 region comprising its egg and yolk, respectively. We will normally write $\mathsf{ey}(X)$ as $\hat{X}$ for notational convenience. We have the following axiom for egg-yolk pairs:

$$\forall X \mathsf{P}(\mathsf{yolkof}(\hat{X}), \mathsf{eggof}(\hat{X})) \ . \tag{39}$$

We then assert the following additional axiom concerning CR:

$$\begin{aligned}\forall X, Y[X \prec Y \rightarrow & \\ & [[\mathsf{PP}(\mathsf{eggof}(\hat{X}), \mathsf{eggof}(\hat{Y})) \wedge \mathsf{P}(\mathsf{yolkof}(\hat{Y}), \mathsf{yolkof}(\hat{X}))] \vee \\ & [\mathsf{P}(\mathsf{eggof}(\hat{X}), \mathsf{eggof}(Y)) \wedge \mathsf{PP}(\mathsf{yolkof}(\hat{Y}), \mathsf{yolkof}(\hat{X}))]]] \ .\end{aligned} \tag{40}$$

This axiom links CR to the predefined RCC5 relations by an *implication*, not an *equivalence*: we do *not* specify that if the specified RCC relations hold between $\mathsf{eggof}(\hat{X})$, $\mathsf{yolkof}(\hat{X})$, $\mathsf{eggof}(\hat{Y})$ and $\mathsf{yolkof}(\hat{Y})$, the CR relation holds between X and Y, but these relations *must* hold for the CR relation to do so. We leave undefined what additional conditions, if any, must be met. This gives us the kind of indefiniteness in the *extent* of vagueness, or *higher-order* vagueness, that intuition demands. Consider the vague region "beside Nick's desk." This can be regarded in $\mathsf{OCregion}$ theory as a $\mathsf{NonCrisp}$ region. There are some precisely defined regions, such as a cube 10cm on a side, 5cm from the right-hand end of Nick's desk, and 50cm from the floor, that are undoubtedly contained within any reasonable complete crisping of this $\mathsf{NonCrisp}$ region. Others, such as a cube 50m on a side centred at the front, top right-hand corner of the desk, contain any such reasonable crisping. These two could correspond to the yolk and egg of an egg-yolk pair constituting the $\mathsf{NonCrisp}$ region "beside Nick's desk," forming a very conservative inner and outer boundary on its possible range of indefiniteness. However, some $\mathsf{OCregions}$ (Crisp

and NonCrisp) lying between this pair would *not* make a reasonable crisping of this region: consider a volume including the 'yolk' of the pair, plus a layer one centimeter deep at the very top of the white. This meets all the conditions for a crisping of the specified OCregion, but is an absurd interpretation of "beside Nick's desk." In general, we need not precisely specify the limits of acceptability. For specific applications, we could add further conditions on acceptable crispings (such as preserving particular topological features or relative proportions in different dimensions), and perhaps assert that (for that application) these conditions are sufficient.

Configuration 1 in Fig.23, given the interpretation of OCregion region theory in terms of egg-yolk pairs of RCC5 regions outlined here, clearly shows a pair of NonCrisp regions such that any pair of complete crispings of the two must be DR . Taking the left egg-yolk pair as representing NonCrisp region X, and the right one NonCrisp region Y:

$$\forall V, W[[V \prec\!\!\prec X \wedge W \prec\!\!\prec Y] \rightarrow \mathsf{DR}(\mathsf{eggof}(\hat{V}), \mathsf{eggof}(\hat{W}))] \tag{41}$$

Similarly, configuration 2 represents a pair of NonCrisp regions such that, for any complete crisping of either, we can choose a complete crisping of the other that is DR from it, and there are also some complete crisping pairs of the two that are PO. (Cohn and Gotts 1994b) shows how each of the forty-six configurations can be distinguished in terms of the possible results of replacing one or both of the egg-yolk pairs with a single region-boundary lying within the white of the egg, a complete crisping of the vague region represented by the egg-yolk.

This way of interpreting OCregion theory explains why we found so many parallels with Simons' mereology. Under the egg-yolk interpretation, an OCregion amounts to a three-way division of u into yolk, white, and non-egg. If we consider a set of all such divisions where no part of space is in the yolk of one division and the non-egg of another, we have a mereological system with all the possible precise boundaries as atoms. Crisping expands yolk and/or non-egg at the expense of the white. One OCregion being a crisping of another is like one individual being a proper part of another because the white of the first is a proper part of the white of the second. We have a plausible candidate for the VCC (Vaguest Common Crisping) of two MA OCregions: the VCC 's yolk could be the sum of the yolks of its two blurrings, its egg the prod of the two blurrings' eggs (which, if the two are MA , must exist as a region). Similarly, the yolk of the CCB (Crispest Common Blurring) of *any* two OCregions might be defined as the prod of their yolks; its egg as the sum of their eggs.

The implications of these identifications remain to be explored. However, the egg-yolk model of the OCregion axioms does appear to provide a straightforward way to define OCregion extensions of the compl, sum, prod and diff functions defined within RCC. Moreover, egg-yolk theory gives us a way to reason with vague regions using the existing mechanism of the RCC calculus.

9. Final Comments

Work is still continuing on RCC and related formalisms, both at Leeds and elsewhere. We are still working on the formal semantics of RCC (Gotts 1996a).[47] RCC, as presented here, does not have sufficient existential axioms. In (Bennett 1996b) some progress is made towards addressing this deficiency and Pratt and Schoop (1997) present a system which is closely related to RCC and shown to be complete with respect to a Euclidean planar model.

There is still further work to do with the axiomatization of conv and indeed in investigating other primitives that would enhance the expressiveness of RCC. We also hope to work further on our approaches to reasoning about indeterminate boundaries. The work on using zero-order logics seems promising, but there is still further work to do on the larger RCC calculi and in formally relating the zero and first-order representations. We are also looking to various applications to drive our work forward.

Although we have mentioned quite a lot of related work, QSR is a growing field and there is not space to do it justice here. Hernández (1994) provides a slightly dated review in a final chapter; also see the survey in (Cohn 1996). The proceedings of COSIT (such as (Frank and Campari 1993, Frank and Kuhn 1995)) contain many related papers. A spatial reasoning web site including a pointer to an online interactive bibliography can be found at:

`http://www.cs.albany.edu/~amit/spatsites.html`

In summary, we have presented a logical calculus for qualitative reasoning about spatial regions, with both a first-order and propositional sub-variant. The system has remarkably few primitives, which is desirable not only from a theoretical viewpoint, but also from an implementational one: one need only implement these few primitives to interface to a perceptual component. RCC provides a rich vocabulary of qualitative shape descriptions and has extensions to handle uncertainty. We have provided some special-purpose reasoning techniques (composition tables and conceptual neighbourhoods) that can be exploited in a qualitative spatial simulator. We have also sketched some possible application areas for RCC.

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Notes

1. Less formal investigations of the semantics of natural language spatial expressions have been conducted by a number of researchers — see e.g. Herskovits' chapter in (Stock 1997).
2. This name is recent and is not used in many of our earlier papers.
3. These and other papers can be obtained *via* the World Wide Web from: `http://www.scs.leeds.ac.uk/spacenet/publications.html`.
4. The acronym RCC and both possible interpretations are due to Antony Galton.
5. 'Mereology' is a term (first used by Leśniewski) to describe the formal theory of part, whole and related concepts.
6. Ladkin (1986) has investigated temporal non convex interval logics. The spatial logic we present below will also allow non-convex spatial entities.
7. This problem has already been noted in a temporal context (Galton 1990).
8. Alternatively, non empty regular closed sets of connected T_3-spaces have been proved to be models for the RCC axiom set (Gotts 1996a).
9. The argument sorts for space are Region and Period, respectively, while the result sort is $\mathsf{Spatial} \sqcup \mathsf{NULL}$. Period is a sort denoting temporal intervals.
10. Quasi, because the lack of a null region means the functions do not form a Boolean algebra.
11. For notational convenience we will sometimes write $x = y$ rather than $\mathsf{EQ}(x, y)$; technically the latter is preferable, since EQ is a relation defined in terms of C rather than true logical equality. However, for readability's sake we will ignore this distinction here.
12. It follows from our definition of the sum function as $x = \mathsf{sum}(y, z) \leftrightarrow \forall w[\mathsf{C}(w, x) \leftrightarrow (\mathsf{C}(w, y) \vee \mathsf{C}(w, z))]$ — consider the case where $y = z$.
13. An interesting question arises: what is so special about RCC8? One answer might be that it is essentially the system that arises (in 1D) if one takes Allen's calculus and ignores the before/after ordering: the thirteen relations collapse to eight, which mirror those of RCC8. However, note that Allen's calculus assumes that all intervals are one piece and further relationships would exist if this were not the case (Ladkin 1986). The 4-intersection model of Egenhofer and Franzosa (1991) also gives rise to exactly eight analogous relations under certain assumptions (such as zero co-dimension). In fact (Dornheim 1995) shows that the interpretation of the RCC8 relations is slightly more general.
14. The corresponding definition in (Gotts 1994b) is faulty.
15. To be employed in their full generality, the predicates $\mathsf{SEPNUM}(x, n)$, $\mathsf{FCON}(x, n)$ and $\mathsf{SBNUM}(x, y, n)$ require the introduction of natural numbers into the system; however, if we only want to use instances of these predicates in cases where n is some given fixed number (as is the case in defining a doughnut) they can always be cashed out, in terms of their (recursive) definitions, to yield complex predicates not containing numbers. Thus the numbers can be regarded as meta-level syntax used to refer to denumerable sequences of predicates. This could be indicated by writing $\mathsf{SEPNUM}_n(x)$, $\mathsf{FCON}_n(x)$ and $\mathsf{SBNUM}_n(x, y)$.
16. Note, however, that this task becomes almost trivial once the $\mathsf{conv}(x)$ primitive is introduced in Section 5.
17. In cases where reasoning about dimensionality becomes important, the RCC system is not very powerful. To remedy this we have proposed a new primitive $\mathsf{INCH}(x, y)$, whose intended interpretation is that spatial entity x *includes a chunk of* y, where the included chunk is of the same dimension as x. The two entities may be of differing (though uniform) dimension. Thus if x is line crossing a 2D region y, then $\mathsf{INCH}(x, y)$ is true, but not vice versa. It is easy to define $\mathsf{C}(x, y)$ in terms of INCH, but not vice versa, so the previous RCC system can be defined as a sub theory. An initial exposition of this theory can be found in (Gotts 1996b). Interestingly, a similar proposal was subsequently made independently by (Galton 1996).

18. As mentioned above when outlining how to define a doughnut, it is possible to describe some non-convex regions using C alone, but it is impossible to describe the holes themselves as regions. Moreover, not all kinds of concave shapes can be distinguished using C alone (for example, depressions in a surface cannot be distinguished). Casati and Varzi (1994) distinguish between hole-realism in which holes are first class objects and hole-adverbialism in which reference to holes is just a "façon-de-parler" and all one really says is that an object is holed (in such and such a way). This is all RC$\{\mathsf{C}, \mathcal{B}\}$ can do. RC$\{\mathsf{C}, \mathcal{B}, \mathsf{conv}\}$ can take a hole-realist position.
19. It should be noted that these axioms are not all independent. It is quite easy to prove that axiom 20 is a consequence of axiom 25 and that axiom 22 is entailed by axiom 23; and it is probable that there are further dependencies.
20. One possible line of attack would be to introduce an alternative primitive, "region y is between regions x and z" (see Tarski's axiomatisation of geometry which uses a point based betweenness primitive (Tarski 1959)) and define conv in terms of this primitive. Linking this primitive to Tarski's point based betweenness relation may provide a way to verify the completeness of the axiomatization.
21. In fact if we were allow regions which are neither finite nor co-finite (having a finite complement) this axiom does not hold, so we get 18 possible refinements of DR.
22. See also their chapter in (Stock 1997) and (Varzi 1996b, Varzi 1996a).
23. In his chapter of (Stock 1997), Frank discusses the general question of ontologies from a consumer's viewpoint.
24. Of course, this lattice allows certain kinds of reasoning involving subsumption and disjointness of relations to be performed efficiently as noted in (Randell and Cohn 1992).
25. Use of the composition table can easily be generalised to handle the case where R_1 and R_2 are disjunctions of relations taken from the set $\{R_i\}$ — we just look up the compositions for all possible pairwise combinations of the disjuncts involved in R_1 and R_2. (Bennett 1994a) discusses various other uses and aspects of composition tables.
26. An interesting question is raised here: under what circumstances is this local consistency checking procedure complete for determining the overall consistency of a set of ground facts, whose relations are constrained by some axiomatic theory? It can be shown that, if a set of RCC8 relations is consistent wrt the RCC8 composition table, then there is a topological model of the set of facts. However, (Grigni et al. 1995) observe that if we constrain regions to be planar and bounded by Jordan curves, an RCC relation set may have no such model, even though it is consistent with the composition table. We have explored question of completeness of composition tables in (Bennett, Isli and Cohn 1997).
27. Our table coincides with that of (Egenhofer 1991), who built an eight relation calculus, which, although based on point set topology, has many similarities to RCC8.
28. Actually, in representing RCC relations in this way it is important to add for each region r an additional entailment constraint $\sim r$ which ensures that the region is non-null (see (Bennett 1994b)).
29. This explains the term *entailment constraint.*
30. Complexity of reasoning about spatial relations is currently far less understood than the (at least superficially) similar domain of temporal relations. Allen's set of thirteen (JEPD) qualitative relations between temporal intervals (Allen 1981, Allen 1983, Allen 1984) and the algebra generated from these relations have been quite extensively explored. The NP-hardness of reasoning about arbitrary disjunctions of the temporal intervals was demonstrated by Vilain and Kautz (1986). Ladkin has investigated the model theory of the relations and their representation within the framework of *relation algebra* (Ladkin 1987, Ladkin and Maddux 1994). A maximal tractable sub-algebra over the Allen relations has been identified by Nebel (1995b).
31. A maximal tractable subsets of disjunctive combinations of the RCC8 relations is identified in (Renz and Nebel 1997).

32. Note that the assumptions about what is continuous behaviour are quite sophisticated here: imagine two regions, one that is two piece and has one component that is an NTPP of the other region and a second component which is DC from the other regions; thus the two regions are PO. If the component which was an NTPP disappeared (a puddle drying in the sun?), then there would be an instantaneous transition from PO to DC! However, we argue that becoming NULL is discontinuous.
33. Exceptions to this are pointed out and considered in (Bennett 1994a).
34. We have also done some work on real time event recognition from image sequences, by means of qualitative event descriptions; our initial work in this area is reported in (Fernyhough, Cohn and Hogg 1996, Fernyhough 1997, Fernyhough, Cohn and Hogg 1997).
35. Here 'maximal' can be stated in qualitative terms by saying that the region is not a proper-part of another sea region which is also part of the convex-hull of the land region.
36. E.g. one might add an order of magnitude representation such as that in (Raiman 1986) or (Mavrovouniotis and Stephanopoulos 1988).
37. A newer, more principled implementation based on the *transition calculus* is described in (Gooday and Cohn 1996a).
38. Fig.19 reveals a subtle difficulty with our analysis of state transition. In the first transition on the second row the food particle crosses the boundary and touches the enzyme all in one step but in fact since the crossing of the boundary happens instantaneously it must precede the coming together of enzyme and food. The distinction between instantaneous and durative changes has been examined by Galton (1995a). One should also realise that because the modelling is done in RCC8 without using conv, the model is not a very accurate representation of reality.
39. Haarslev (1996) has also presented a spatial calculus for similar purposes.
40. We termed this representation the the 'egg-yolk' calculus, for obvious reasons, and will meet it again when describing an extension to RCC to handle regions with indeterminate boundaries below.
41. We are sceptical about the merits of 'fuzzy' approaches to indeterminacy, believing that their use of real number indices of degrees of membership and truth are both counterintuitive and logically problematic. We have no space to argue this controversial viewpoint here; see (Elkan 1994) and responses for arguments on both sides.
42. Note that we have addressed only the question of modelling indeterminate boundaries rather than indeterminate position.
43. We will use upper-case italic letters for variables ranging over OCregions. These are optionally crisp regions, which may be crisp or not.
44. The case of RCC8 is addressed in (Cohn and Gotts 1996b).
45. Each cluster of Fig.24 represents one of these conceptual neighbourhoods.
46. Clementini and Di Felice (1994) have also produced a very similar analysis based on Egenhofer's 9-intersection method, though they omit two of the forty six relations, which they do not believe are possible in their domain. They apply their calculus of regions with broad boundaries to a number of situations (Clementini and Di Felice 1997) including reasoning about discrete spaces, convex hulls and minimum bounding rectangles.
47. Asher and Vieu (1995) have provided a formal semantics for Clarke's system.

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